Translation with Quantifiers

The Step-by-Step Method

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Announcements

11.03

1. The midterms are graded, they will be handed back on Tuesday
   - Check on grades through Bb this weekend
2. Additional HW scores should be available too

Outline

The 4 Aristotelian Forms

Review

The Aristotelian Forms and Their Translations

All A's are B's	$\forall x\,(A(x) \rightarrow B(x))$
Some A's are B's	$\exists x\,(A(x) \wedge B(x))$
No A's are B's	$\forall x\,(A(x) \rightarrow \neg B(x))$
Some A's are not B's	$\exists x\,(A(x) \wedge \neg B(x))$

Subjects and Objects
Some Terminology

- Some predicates like *love* relate two things:
  (1) *Kay loves Jay*
- When you have a predicate that relates two things, it's helpful to have some terminology to distinguish those two things
- *Kay* is the subject
- *Jay* is the object
- Intuitively, the subject is what the sentence is primarily about

Roaming Quantifiers
In Object Position

- So far, we've only considered sentences with quantifiers in subject-position:
  (2) *Every cube is in front of **b***
- What about when you have a quantifier in object-position?
  (3) ***b** is in front of everything*
- Just stick ∀ out in front of the predicate, and 'quantify into' the object position

$$\forall x\, \mathsf{FrontOf}(b, x)$$

Roaming Quantifiers
More on Object Position

- Okay, but what happens when the quantifier in object position is restricted
  (4) ***b** is in front of every cube*
- You have to move its restrictor out front too:
  (4′) $\forall x\, (\mathsf{Cube}(x) \to \mathsf{FrontOf}(b, x))$
- This holds for multiply restricted ones too:
  (5) ***b** is in front of every small cube*
  Translates as:
  (5′) $\forall x\, ((\mathsf{Cube}(x) \land \mathsf{Small}(x)) \to \mathsf{FrontOf}(b, x))$

Roaming Quantifiers
Some More Examples

(6) shows that you move the restrictors to the left of the predicate, but no further!

(6)
a. *It's not the case that **b** is a large cube*
b. $\neg\exists y\, (\mathsf{Large}(y) \land \mathsf{Cube}(y) \land b = y)$

(7)
a. *It's not the case that something is a large cube*
b. $\neg\exists y\, (\mathsf{Large}(y) \land \mathsf{Cube}(y) \land \exists x\, x = y)$

(8)
a. *Everything between **c** and **b** is **a***
b. $\forall x\, (\mathsf{Between}(x, c, b) \to x = a)$

(9)
a. *Everything between **c** and **b** is a cube*
b. $\forall x\, (\mathsf{Between}(x, c, b) \to \exists y\, (\mathsf{Cube}(y) \land x = y))$

A Systematic Method
For Translating Mixed Quantifiers

- Translating simple quantificational sentences into FOL is hard enough
- Once we consider sentences with multiple and mixed quantifiers, things get even harder
- To address this situation we are going to learn a systematic method for translating quantificational sentences
- We'll first go through an application of the method and then state abstract what the method is

The Method
Learning by Example

(10) *Every cube is to the left of a tetrahedron*

- First, note (10)'s general form: *Every A is B*
- So, our translation will have the form:
  (10′) $\forall x\, [A(x) \rightarrow B(x)]$
- We just need to find $A(x)$ and $B(x)$
- $A(x) = Cube(x)$, but what is $B(x)$?
- Something like: x *is-to-the-left-of-a-tetrahedron*
  - This predicate translates as: $\exists y\, (Tet(y) \wedge LeftOf(x, y))$
  - So, $B(x) = \exists y\, (Tet(y) \wedge LeftOf(x, y))$
- Finally, we just plug $A(x)$ and $B(x)$ into (10′):
  (10″) $\forall x\, [Cube(x) \rightarrow \exists y\, (Tet(y) \wedge LeftOf(x, y))]$

The Method
In the Abstract

The Step-by-Step Translation Method

1. Determine the general form of the sentence
   - E.g. *Every A is B, No A is B*
2. Write the *skeleton* for that form
   - E.g. $\forall x\, [A(x) \rightarrow B(x)], \forall x\, [A(x) \rightarrow \neg B(x)]$
3. Find the parts of the skeleton:
   - E.g. find $A(x)$ and $B(x)$
   - If the parts are complex, start with an informal approximation
   - If the parts themselves contain mixed or multiple quantifiers, repeat this method on them
4. Plug the parts into the skeleton

The Method
A Second Example

(11) *Some tetrahedron is in front of every small cube*

1. The Form: *Some A is B*
2. The Skeleton: $\exists x\, [A(x) \wedge B(x)]$
3. Find the parts:
   - $A(x) = Tet(x)$, what about $B(x)$?
   - $B(x)$ is complex, so informally approximate: x *is-in-front-of-every-small-cube*
   - Now translate $B(x)$: $\forall y\, ((Small(y) \wedge Cube(y)) \rightarrow FrontOf(x, y))$
4. Fill in the skeleton:

(11′) $\exists x\, [Tet(x) \wedge \forall y\, ((Small(y) \wedge Cube(y)) \rightarrow FrontOf(x, y))]$

The Method
How to Write it Down

(11) *Some tetrahedron is in front of every small cube*

1. Recognize the form and write the appropriate skeleton:
   (11a) $\exists x\,[\mathsf{A}(x) \land \mathsf{B}(x)]$
2. Fill incrementally, using approximation where necessary:
   (11b) $\exists x\,[\mathsf{Tet}(x) \land \mathsf{B}(x)]$
   (11c) $\exists x\,[\mathsf{Tet}(x) \land x$ *is-in-front-of-every-small-cube*$]$
3. Once you've arrived at approximations with a single quantifier, translate them and plug them in:

(11′) $\exists x\,[\mathsf{Tet}(x) \land \forall y\,((\mathsf{Small}(y) \land \mathsf{Cube}(y)) \to \mathsf{FrontOf}(x,y))]$

Yet Another Example
A Harder One

(12) *Some cube with nothing in front of it has something in back of it*

1. The form is *Some A is B*, so its skeleton is:
   (14a) $\exists x\,[\mathsf{A}(x) \land \mathsf{B}(x)]$
2. $\mathsf{A}(x)$ and $\mathsf{B}(x)$ are complex, so we start with approximations:

(14b) $\exists x\,[\mathsf{A}(x) \land$ *something-is-in-back-of* $x]$

(14c) $\exists x\,[\mathsf{A}(x) \land \exists z\,\mathsf{BackOf}(z,x)]$

(14d) $\exists x\,[(\mathsf{Cube}(x) \land$ *nothing-is-in-front-of* $x) \land \exists z\,\mathsf{BackOf}(z,x)]$

(14′) $\exists x\,[(\mathsf{Cube}(x) \land \forall y\,\neg\mathsf{FrontOf}(y,x)) \land \exists z\,\mathsf{BackOf}(z,x)]$

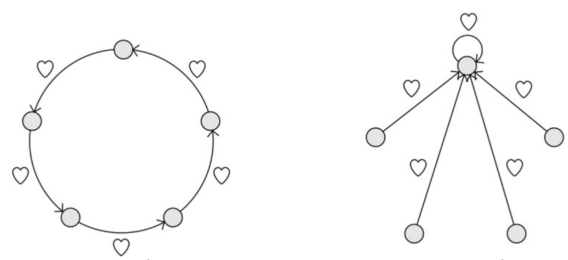
In Class Exercise

Exercise **11.39**

The Method
Particularity

(13) *Every one loves a particular someone*

- Which picture does this describe?



No! Yes!

- So, we translate (13) as:
  (13′) $\exists y\,\forall x\,\mathsf{Loves}(x,y)$

The Method
Some Questions

(13) *Every one loves a particular someone*

(13′) $\exists y\, \forall x\, \mathsf{Loves}(x, y)$

- In (13) the order of appearance is *Universal-Existential*
- In (13′) the order is *Existential-Universal*
- What gives?
- The word *particular* signals that the existential takes *widest scope*
- In general $\exists\forall$ sentences describe some particular thing being related to everything
- $\forall\exists$ sentences describe every thing being related to some thing or other

The Method
A More Advanced Application

(14) *Every cube is the same size as a particular tetrahedron*

- The form is *Every A is B*, so we start from the appropriate skeleton:

  (14a) $\forall x\, (\mathsf{A}(x) \to \mathsf{B}(x))$

  (14b) $\forall x\, (\mathsf{Cube}(x) \to \mathsf{B}(x))$

  (14c) $\forall x\, (\mathsf{Cube}(x) \to$ x *the same size as-a-particular-tet*)

  (14d) $\forall x\, (\mathsf{Cube}(x) \to \mathsf{SameSize}($x, *a-particular-tet*))

- We know *particular* makes existentials take wide scope, so the next step from (14d) is:

  (14′) $\exists y\, [\mathsf{Tet}(y) \land \forall x\, (\mathsf{Cube}(x) \to \mathsf{SameSize}(x, y))]$

Ambiguity
A Catch

- Our step-by-step method works wonderfully in most cases
- But, there are some things you have to be wary of when translating from English to FOL
- One of them is ambiguity

Ambiguity

(15) *Every minute a man is mugged in New York City*

  - **Joke**: we are going to meet the poor guy tonight

- We generally interpret (15) as:

  (15a) $\forall x\, [\mathsf{Min}(x) \to \exists y\, (\mathsf{Man}(y) \land \mathsf{MuggedIn}(y, x, nyc))]$

  - *For every minute **x**, there is at least one man **y** such that **y** is mugged at **x** in NYC*

- But the joke plays on the fact that (15) also seems to leave open the interpretation:

  (15b) $\exists y\, [\mathsf{Man}(y) \land \forall x\, (\mathsf{Min}(x) \to \mathsf{MuggedIn}(y, x, nyc))]$

  - *There is at least one man **y** such that for every minute **x**, **y** is mugged at **x** in NYC*

Ambiguity

What it is and Why it Matters

- In general, a sentence is *ambiguous* when it has two or more different interpretations
- Sentences involving quantifiers have a preferred interpretation, but a second interpretation is often possible
- Sentences of FOL are not ambiguous
- So, when you translate an English sentence into FOL you will sometimes have to think about which possible interpretation of that sentence you should be capturing
- In FOL, these differences generally amount to different quantifier orderings

Ambiguity

Another Example

(16) *Every cube is the same size as a dodecahedron*

- Does (16) say that every cube is the same size as some dodec or other?
- Or does it say that there is a particular dodec which every cube is the same size as?
- Let's solidify the difference between the two claims in Tarski's World