# The Puzzles of Deontic Logic

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## 1 Introduction

**Primary Reading**: McNamara (2010)

- Deontic logic is concerned with patterns of consequence and consistency between claims involving various modal verbs and auxiliaries.
- To illustrate the idea, consider various statements considering regulations at a state park.

(1) a. Camping is permitted.
  b. Hunting is forbidden/prohibited.
  c. Registration is required/obligatory.
  d. Ties are optional.

(2) a. You may camp./You can camp.
  b. You may not hunt./You must not hunt./You cannot hunt.
  c. You must register.
  d. You should register.
  e. You ought to register.

**Today's Question** How far can one get with an analysis of these terms in modal logic?

### 1.1 Modal Deontic Logic

- Modal logic assigns propositions to sentences using four elements:
  - A *valuation* $v$ of atomic sentences (familiar from truth-functional logic)
    - ▶ Maps every atomic sentence to either 1 or 0
  - A space of *possible worlds* $W$
  - An *accessibility relation* $R$
    - ▶ $R(w, w')$ just in case $w'$ is possible wrt $w$
      - ▷ Deontic interpretation: $R(w, w')$ means that $w'$ is *permissible* with respect to the laws in $w$

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    - ▶ For now, we won't place any constraints on $R$
    - ▶ Notation: $R(w) := \{w' \mid R(w, w')\}$
  - *Models* $\mathcal{M} = \langle\langle R, W\rangle, v\rangle$
    - ▶ A model says what the facts are like and how our sentences hook up to them
  - **Modal Logic Semantics**

(1) $[\![\mathsf{A}]\!]^{\mathcal{M}} = \{w \mid v(\mathsf{A}, w) = 1\}$

(2) $[\![\neg\phi]\!]^{\mathcal{M}} = W - [\![\phi]\!]^{\mathcal{M}}$

(3) $[\![\phi \wedge \psi]\!]^{\mathcal{M}} = [\![\phi]\!]^{\mathcal{M}} \cap [\![\psi]\!]^{\mathcal{M}}$

(4) $[\![\Box\phi]\!]^{\mathcal{M}} = \{w \mid R(w) \subseteq [\![\phi]\!]^{\mathcal{M}}\}$

(5) $[\![\Diamond\phi]\!]^{\mathcal{M}} = \{w \mid R(w) \cap [\![\phi]\!]^{\mathcal{M}} \neq \varnothing\}$

- **Consequence**: $\phi_1, \ldots, \phi_n \vDash \psi \iff$ For every $\mathcal{M}$: $[\![\phi_1]\!]^{\mathcal{M}} \cap \cdots \cap [\![\phi_n]\!]^{\mathcal{M}} \subseteq [\![\psi]\!]^{\mathcal{M}}$
- **Truth**: $\mathcal{M}, w \vDash \phi \iff w \in [\![\phi]\!]^{\mathcal{M}}$
- **Logical Truth**: $\vDash \phi \iff \forall \mathcal{M}, w : \mathcal{M}, w \vDash \phi$
- The only plausible choices for analyzing the terms in (1) and (2):

| English | MDL |
|---|---|
| *A is permitted* | $\Diamond\mathsf{A}$ |
| *A is forbidden/prohibited* | $\Box\neg\mathsf{A}/\neg\Diamond\mathsf{A}$ |
| *A is required/obligatory* | $\Box\mathsf{A}$ |
| *A is optional* | $\Diamond\mathsf{A} \wedge \Diamond\neg\mathsf{A}$ |
| *You may/can A* | $\Diamond\mathsf{A}$ |
| *You may/can/must not A* | $\neg\Diamond\mathsf{A}/\Box\neg\mathsf{A}$ |
| *You must A* | $\Box\mathsf{A}$ |
| *You should/ought to A* | $\Box\mathsf{A}$ |

## 2 Puzzles of Deontic Logic

### 2.1 The Logical Necessity of Obligation

- Regardless of how $R$ is constrained, the following is true:
  - If $\phi$ is a logical truth, then $\Box\phi$ is a logical truth
    If $\vDash \phi$ then $\vDash \Box\phi$
- This means that the following are logical truths on the MDL analysis:

(3) a. Running or not running is required.
  b. You must run or not run.

c. You should run or not run.

d. You ought to run or not run.

e. You must not run and not run.

- It also predicts that the following cannot be true:

(4) There is nothing you are required to do.

- McNamara (2010: §4.2) discusses attempts to eliminate this feature from the semantics
- But might a pragmatic analysis do (and be more appropriate)?

### 2.2 The Good Samaritan (Prior 1958)

- The version from Prior (1958):
  - Let's hope (6) does not follow from (5):

    (5) It is required that Jones help Smith, who has been robbed

    (6) It is required that Smith be robbed
  - The following seems to be a necessary truth:
    - Jones helps Smith, who has been robbed if and only if Jones helps Smith and Smith has been robbed.
  - But then, (5) translates as $\Box(\mathsf{H} \wedge \mathsf{R})$
  - However, $\Box(\mathsf{H} \wedge \mathsf{R}) \vDash \Box\mathsf{R}$, so (6) follows from (5)!
- The conditional version from Kratzer (1991: §8):

(7) a. If a murder occurred, the jury is required to convene

b. The jury is permitted not to convene

c. A murder occurred

  - First analysis: $\mathsf{M} \supset \Box\mathsf{J}$
    - So it follows by modus ponens that $\Box\mathsf{J}$
    - But this contradicts (7): $\Diamond\neg\mathsf{J}$
      - This seems to be a counter-example to modus ponens!
    - Putting this actual contradiction aside, it follows that at every permissible world, $\mathsf{J}$ is true
    - Question: did a murder also occur in this world?
    - Yes, but then it follows that $\Diamond\mathsf{M}$!
  - Second analysis: $\Box(\mathsf{M} \supset \mathsf{J})$
    - It does not follow from this and $\mathsf{M}$ that $\Box\mathsf{J}$, so modus ponens is safe
    - But together with $\Box\neg\mathsf{M}$ it follows that $\Box\mathsf{J}$ and the tension resurfaces



- Making the conditional version harder:

(8) a. If Bob murders someone, the executioner is required to kill Bob

d. Bob murdered someone

  - First analysis: $\mathsf{M} \supset \Box\mathsf{K}$
    - So it follows by modus ponens that $\Box\mathsf{K}$
    - It follows that at every permissible world, $\mathsf{K}$ is true
    - Question: did a murder also occur in this world?
    - If Yes, it follows that $\Diamond\mathsf{M}$!
    - If No, the executioner killed Bob without cause, so there was a murder in this permissible world
    - So either way there is murder in permissible worlds
  - Second analysis: $\Box(\mathsf{M} \supset \mathsf{K})$
    - But together with $\Box\neg\mathsf{M}$ it follows that $\Box\mathsf{K}$
    - As before, there must be murder in some permissible world
      - Either Bob murdered someone and the executioner killed Bob
      - Or the executioner killed Bob, who was innocent!
- Kratzer (1991: §8) claims to have a solution to the conditional version
  - How would it go?

### 2.3 Sartre's Dilemma (Lemmon 1962)

- Last week I promised Ann I'd have a drink with her on Easter, and I promised Bill I would not drink on Sundays.
- Obviously, I didn't realize that Easter is on Sunday.
- When I did, I realized that the following were now both true:

(9) I'm obligated to drink with Ann on Easter Sunday

(10) I'm obligated to not drink with Ann on Easter Sunday

Or, similarly:

(11) I ought to drink with Ann on Easter Sunday

(12) I'm ought to not drink with Ann on Easter Sunday

- But in MDL this is a necessary falsehood: $\Box\mathsf{D} \wedge \Box\neg\mathsf{D}$
- While (9) and (10) are true, is the following?

(13) I'm obligated to drink and to not drink with Ann on Easter Sunday

- What do people think about the *required* and *must* versions?

(14) I'm required to drink with Ann on Easter Sunday

(15) I'm required to not drink with Ann on Easter Sunday

Or, similarly:

(16) I must drink with Ann on Easter Sunday

(17) I must not drink with Ann on Easter Sunday

- In the semantics literature, modals like *ought* are called *weak necessity modals*
  - They are thought to have semantics of 'weak necessity' (Kratzer 1991)
  - For every not-$p$-world, there is a $p$-world at least as good as it; and: not vice versa
  - But even on this view it is impossible for $\mathsf{Ought}\,\mathsf{p}$ and $\mathsf{Ought}\,\neg\mathsf{p}$ to be true!

### 2.4 Plato's Dilemma (Lemmon 1962)

(See also Marcus 1980)

- Last week I promised Ann I'd dine with her on Easter, and I promised Bill I would dine with him next Sunday.
- Unfortunately, I didn't realize that Easter was next Sunday and, Ann and Bill refuse to dine together.
- When I did, I realized that the following were now both true:

(18) I'm obligated to dine with Ann on Easter Sunday

(19) I'm obligated to dine with Bill on Easter Sunday

Or, similarly:

(20) I ought to dine with Ann on Easter Sunday

(21) I'm ought to not drink with Ann on Easter Sunday

- But in MDL this alone isn't a necessary falsehood: $\Box\mathsf{A} \wedge \Box\mathsf{B}$
  - But if we add the premise that dining with both is prohibited, contradiction results
    - $\neg\Diamond(\mathsf{A} \wedge \mathsf{B})$
- What do people think about the *required* and *must* versions?

(22) I'm required to dine with Ann on Easter Sunday

(23) I'm required to dine with Bill on Easter Sunday

Or, similarly:

(24) I must dine with Ann on Easter Sunday

(25) I must dine with Bill on Easter Sunday

- Kratzer's weak necessity operators again don't seem to do the trick



### 2.5 Chisholm's Paradox (Chisholm 1963)

- The Paradox begins with this clearly consistent set of claims:

(26) a. Jones ought to go (assist his neighbors)
b. It ought to be that if Jones goes, he tells them he's coming
c. If Jones doesn't go, he ought to not tell them he's coming
d. As a matter of fact, Jones didn't go

(27) a. Jones is required to go (assist his neighbors)
b. It is required that if Jones goes, he tells them he's coming
c. If Jones doesn't go, he is required to not tell them he's coming
d. As a matter of fact, Jones didn't go

- Simplest translation into MDL:

(28) a. $\Box\mathsf{G}$
b. $\Box(\mathsf{G} \supset \mathsf{C})$
c. $\neg\mathsf{G} \supset \Box\neg\mathsf{C}$
d. $\neg\mathsf{G}$

- These are inconsistent!
  - In MDL, $\Box(\mathsf{G} \supset \mathsf{C})$ entails $\Box\mathsf{G} \supset \Box\mathsf{C}$
  - Then from (28a) $\Box\mathsf{C}$ follows by modus ponens
  - But from (28c) and (28d) $\Box\neg\mathsf{C}$ follows by modus ponens
- The inconsistency can be blocked by either always wide-scoping $\Box$ in conditionals, or always narrow-scoping
  - Option 2:

(29) a. $\Box\mathsf{G}$
b. $\Box(\mathsf{G} \supset \mathsf{C})$
c. $\Box(\neg\mathsf{G} \supset \neg\mathsf{C})$
d. $\neg\mathsf{G}$

  - Option 3:

(30) a. $\Box\mathsf{G}$
b. $\mathsf{G} \supset \Box\mathsf{C}$
c. $\neg\mathsf{G} \supset \Box\neg\mathsf{C}$
d. $\neg\mathsf{G}$

- The problem with these translations:
  - They don't capture the fact that the claims in (26) are *logical independent*
- Option 2:
  - From $\Box\mathsf{G}$, $\Box(\neg\mathsf{G} \supset \neg\mathsf{C})$ follows
- Option 3:
  - From $\neg\mathsf{G}$, $\mathsf{G} \supset \Box\neg\mathsf{C}$ follows
- In general, the scoping strategy seems silly, since (31) and (32) sound equivalent

(31) It is required that if Jones goes, he tells them he's coming

(32) If Jones goes, it is required that he tells them he's coming

- The fact that these aren't equivalent in MDL is itself a problem!
- Recall that on Kratzer's approach, conditionals are just modals with explicit restrictors
  - Does her account solve Chisholm's Paradox?
  - It makes (31) and (32) equivalent, since they come out as the same logical form:
    - $\mathsf{Req(G)(C)}$
  - This means: if you add $[\![\mathsf{G}]\!]$ to the modal base, all the best worlds are C-worlds
    - It is consistent with this that according to the actual modal base, none of the best worlds are C-worlds!

### 2.6 Forrester's Paradox (Forrester 1984)

- Consider these consistent sets of claims:

(33) a. It is forbidden for John to kill his mother
b. If John does kill his mother, he is obligated to kill her gently
c. John killed his mother

(34) a. John must not kill his mother
b. If John does kill his mother, he must kill her gently
c. John killed his mother

- Their translations are inconsistent in MDL:
  - $\Box\neg\mathsf{K}$, $\mathsf{K} \supset \Box\mathsf{G}$, $\mathsf{K}$
    - By modus ponens, we have $\Box\mathsf{G}$.
    - If Jones kills his mother gently, then he kills her.
    - So: $\mathsf{G} \vDash \mathsf{K}$
    - If $\mathsf{G} \vDash \mathsf{K}$ and $\Box\mathsf{G}$, then $\Box\mathsf{K}$
    - But this contradicts $\Box\neg\mathsf{K}$!
  - The culprit: substitution of logical consequences under $\Box$
- Again, Kratzer's theory seems to do interesting work:
  - (34a) says that according to the actual modal base and ordering source, none of the best worlds are K-worlds
  - (34b) says that after adding G to the modal base, all the best worlds are K-worlds
  - It does not then follow from K that $\Box\mathsf{G}$, since it does not follow that according to the actual modal base, all of the best worlds are G-worlds

### 2.7 *Must* versus *Ought* (McNamara 1996)

- This sentence is consistent:

(35) You may skip the talk, but you ought to come

  - But on the only choice in MDL is inconsistent:
    - $\Diamond\neg\mathsf{C} \wedge \Box\mathsf{C}$
- This sentence is not consistent:

(36) You may skip the talk, but you must come

- Conclusion: *must* and *ought* have different quantificational strength
- An analysis in terms of Kratzerian weak necessity seems promising, but see also von Fintel & Iatridou (2008)
  - Who, incidentally, claim that necessity and weak necessity collapse with the limit assumption
- Is there a parallel in the modal verb category?

(37) You are permitted to skip the talk, but it is preferred that you attend

### 2.8 Ross's Paradox (Ross 1941)

- Ross was interested in imperatives, but his observation extends to modals
- Neither b case seems to follow from the a case:

(38) a. You may camp
b. You may camp or hunt

(39) a. You must register
b. You must register or sleep in the outhouse

- But in MDL, $\Diamond\mathsf{C} \vDash \Diamond(\mathsf{C} \vee \mathsf{H})$ and $\Box\mathsf{R} \vDash \Box(\mathsf{R} \vee \mathsf{O})$
- In general, if $\phi \vDash \psi$ it follows from $\Diamond\phi$ that $\Diamond\psi$ and it follows from $\Box\phi$ that $\Box\psi$

- In this case, it follows from $\mathsf{C}$ that $\mathsf{C} \vee \mathsf{H}$
- Actually, matters are worse, since neither of the b cases seem to follow

(40) a. You may camp ($\Diamond\mathsf{C}$)

b. You may camp or you may hunt ($\Diamond\mathsf{C} \vee \Diamond\mathsf{H}$)

(41) a. You must register ($\Box\mathsf{R}$)

b. You must register or you must sleep in the outhouse ($\Box\mathsf{R} \vee \Box\mathsf{O}$)

- But this seems to be a problem for a Boolean account of *or* in general!
  - Admittedly, it is about the interaction...

### 2.9 Free Choice Permission (Kamp 1973)

(McNamara 2010 incorrectly attributes this to Ross 1941)

- The b and c cases seem to follow from the a case:

(42) a. You may camp or hunt

b. You may camp

c. You may hunt

- Does this follow?

(43) You may camp and hunt

  - It seems not, sense you can say *You may camp or hunt, but not both*
- Obviously, we do not have $\Diamond(\mathsf{H} \vee \mathsf{C}) \vDash \Diamond\mathsf{H}$ in $\mathsf{MDL}$

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