Epistemic Modality and Dynamic Semantics

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1 Review

- We're trying to understanding the meaning of epistemic modals:

  (1) The light in the next room **might** be on
- Since modals like *might* aren't truth-functional, our first stop was **modal logic**
  - Here, the meaning of a sentence is identified with a set of possible worlds
    - Intuitively, the worlds compatible with its truth
    - We'll call sets of worlds **propositions**
  - This captures the idea that sentences provide **information**
    - The world is some of *these* ways, and none of *those*
- Modal logic assigns propositions to sentences using four elements:
  - A *valuation* v of atomic sentences (familiar from truth-functional logic)
    - Maps every atomic sentence to either 1 or 0
  - A space of *possible worlds* W
  - An *accessibility relation* R
    - $R(w, w')$ just in case w' is possible wrt w
  - *Models* $\mathcal{M} = \langle\langle R, W\rangle, v\rangle$
    - A model says what the facts are like and how our sentences hook up to them
  - **Modal Logic Semantics**

    (1) $[\![\mathsf{A}]\!]^{\mathcal{M}} = \{w \mid v(\mathsf{A}, w) = 1\}$

    (2) $[\![\neg\phi]\!]^{\mathcal{M}} = W - [\![\phi]\!]^{\mathcal{M}}$

    (3) $[\![\phi \wedge \psi]\!]^{\mathcal{M}} = [\![\phi]\!]^{\mathcal{M}} \cap [\![\psi]\!]^{\mathcal{M}}$

    (4) $[\![\Box\phi]\!]^{\mathcal{M}} = \{w \mid \{w' \mid R(w, w')\} \subseteq [\![\phi]\!]^{\mathcal{M}}\}$

    (5) $[\![\Diamond\phi]\!]^{\mathcal{M}} = \{w \mid \{w' \mid R(w, w')\} \cap [\![\phi]\!]^{\mathcal{M}} \neq \varnothing\}$

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- **Consequence**: $\phi_1, \ldots, \phi_n \vDash \psi \iff$ For every $\mathcal{M}$: $[\![\phi_1]\!]^{\mathcal{M}} \cap \cdots \cap [\![\phi_n]\!]^{\mathcal{M}} \subseteq [\![\psi]\!]^{\mathcal{M}}$
  - ▶ $\mathcal{M} = \langle\langle R, W\rangle, v\rangle$
- **Truth**: $\mathcal{M}, w \vDash \phi \iff w \in [\![\phi]\!]^{\mathcal{M}}$

1.1 Contextualism

- **Problem 1**: this does not capture the context-sensitivity of modals
  - Given our current information, (1) seems true
  - Now suppose we all walk over to the next room and observe that the light is off
  - Given this information, (1) seems false
  - Modal logic doesn't capture this: settling the model settles R once and for all
    - But if R remains constant both *might*-claims express the same proposition
- **Kratzer's Solution**: contextualism
  - Following Kaplan (1978) on indexicals (*I*, *here*, *now*, etc.), Kratzer (1977, 1981, 1991) proposes to treat modals as context-sensitive
  - The meaning of ϕ, $[\![\phi]\!]^{\mathcal{M}}$ is a function from contexts to propositions
    - Kaplan calls this a *character* and calls propositions *contents*
  - We can write $[\![\phi]\!]^{\mathcal{M}}_c$ to mean the proposition expressed by ϕ in c
  - What is a context (c)?
    - A location where an utterance place
      - ▷ A world w_c
      - ▷ A time t_c
      - ▷ An agent a_c
      - ▷ Kratzer: a modal base B_c
        - ⋆ Available info about what is possible with respect to each world w
        - ⋆ So $B(w)$ is the set of worlds possible with respect to w
    - So $c = \langle w_c, t_c, a_c, B_c\rangle$
  - **Kratzer's Semantics**: $[\![\Diamond\phi]\!]^{\mathcal{M}}_c = \{w \mid B_c(w) \cap [\![\phi]\!]^{\mathcal{M}}_c \neq \varnothing\}$
    - **Consequence**: $\phi_1, \ldots, \phi_n \vDash \psi \iff$ For every $\mathcal{M}, c$: $[\![\phi_1]\!]^{\mathcal{M}}_c \cap \cdots \cap [\![\phi_n]\!]^{\mathcal{M}}_c \subseteq [\![\psi]\!]^{\mathcal{M}}_c$
    - **Truth**: $\mathcal{M}, w, c \vDash \phi \iff w \in [\![\phi]\!]^{\mathcal{M}}_c$
  - The point: in different contexts $\Diamond\phi$ will express different propositions
    - Yet, $\mathcal{M}$ stays fixed!
- According to Kratzer's contextualism, the proposition expressed by a modal is determined by the modal base available in the context of utterance

- This contextualist position came to be the standard in philosophy too
  - Only recently have philosophers challenged it with **relativism**

1.2 Relativism

- **Problem 2**: eavesdropping cases are argued to involve epistemic modals that are not interpreted with respect to the modal base of the *utterance context*, but rather the *assessment context*
  - Bond case Egan (2007):
    - Bond and Leiter are in London listening to a bug Bond planted in a conference room in SPECTRE's headquarters in the Swiss Alps. Bond left behind some misleading evidence pointing to his presence in Zürich. Blofeld finds the evidence, takes it to be genuine, and turns to his No.2:

      (2) Bond might be in Zürich
    - No.2, sensibly replies "That's true."
    - But, now consider Leiter, hearing this all from London. He's not at all inclined to say "That's true." when *he* hears (2) from Blofeld, even though Leiter knows that it is compatible with what Blofeld knows.
  - The relativist take:
    - When assessed by No.2, (2) is true
    - When assessed by Leiter, (2) is false
  - **Relativist Semantics**: $[\![\Diamond\phi]\!]_c^{\mathcal{M}} = \{\langle w, j\rangle \mid B_j(w) \cap [\![\phi]\!]_c^{\mathcal{M}} \neq \varnothing\}$
    - j is the judge of the **assessment context**
    - B_j is j's modal base
    - **Relativist Truth**: $\mathcal{M}, c, w, j \vDash \phi \iff \langle w, j\rangle \in [\![\phi]\!]_c^{\mathcal{M}}$
    - Weatherson prefers *content relativism*:
      - $[\![\Diamond\phi]\!]_{c,a}^{\mathcal{M}} = \{w \mid B_{j_a}(w) \cap [\![\phi]\!]_c^{\mathcal{M}} \neq \varnothing\}$
      - This leaves truth and content as is
- Reply: von Fintel & Gillies (2008)
  - The data is less clear than assumed and more heterogenous
  - Sometimes an agent can 'stick to their guns'
  - Mordecai can agree with Pascal's less informed claims, etc.

2 Expressivism

- **Problem 3**: epistemic contradictions don't embed (Yalcin 2011, 2007)

  - This sounds terrible:

    (3) # It's raining and it might not be raining
  - Why? A descriptivist-friendly explanation:
    - Well, (4) is also bad

      (4) It's raining and I don't know it's raining
    - It is Moore-paradoxical: it asserts something whose truth undermines the epistemic grounds for the utterance itself
      - Fashionable analysis: knowledge is the norm of assertion (Williamson 1996)
    - But on the descriptivist views (3) says basically the same thing as (4)
      - So let's just give the same (pragmatic) explanation!
  - Yalcin turns this attempt against the descriptivist
    - When embedded, the *might* version remains bad, but the *know* version doesn't

      (5) a. # Suppose it's raining and it might not be raining
          b. Suppose it's raining and I don't know that it's raining

      (6) a. # If it's raining and it might not be raining...
          b. If it's raining and I don't know that it's raining...
    - The pragmatic analysis of (3) does not extend to (5) or (6)
      - That analysis was based on the idea that both conjuncts were *asserted*
      - But in (5) or (6), they aren't!
- Possible solution (my spin on the 'diagonalization' view in Yalcin 2007):[1]
  - Modals aren't used to discriminate worlds, but what's possible with respect to them
    - So modal propositions need to not only discriminate between w and w', but between the *possibility spheres* $R(w)$ and $R'(w)$
      - Think of R as an accessibility relation and $R(w)$ as the set of all worlds w' s.t. $R(w, w')$
      - Like a centered world, but centered using 'accessibility-spheres'
    - This can be accomplished by taking propositions to be sets of world, proposition pairs of a specific kind: $\langle w, R(w)\rangle$
  - **New Semantics**: $[\![\Diamond\phi]\!]_c^{\mathcal{M}} = \{\langle w, R(w)\rangle \mid \exists w' \in R(w) : \langle w', R(w)\rangle \in [\![\phi]\!]_c^{\mathcal{M}}$
    - Note: this does not mean that $\Diamond\phi$ isn't true in $\langle w, R(w')\rangle$ where $w \neq w'$ since it

[1] The view I formulate is a more reasonable opponent for Yalcin than the one he considers. It is factualist, does not need any privileged notion of the information state of the context, does not require fancy footwork with distinguishing between compositional semantic value and content nor diagonalization and captures epistemic contradiction.

may be that $R(w) = R(w')$

- This does not assume the R's are provided by context, but it is compatible with a view that context provides an R or a set of R's

- **Goal**: explain epistemic contradictions in terms of compositional semantic value
- The explanation of (3) is partly pragmatic
- A new *pragmatic* notion:

Acceptance ϕ is accepted in $R(w)$ iff $\forall w' \in R(w) : \langle w', R(w)\rangle \in [\![\phi]\!]_c^{\mathcal{M}}$

- For $\mathsf{p} \wedge \mathsf{Might}(\neg \mathsf{p})$, this requires $\forall w' \in R(w) : \langle w', R(w)\rangle \in [\![\mathsf{p} \wedge \mathsf{Might}(\neg\mathsf{p})]\!]_c^{\mathcal{M}}$.
  - The first conjunct requires: $\forall w' \in R(w) : \langle w', R(w)\rangle \in [\![\mathsf{p}]\!]_c^{\mathcal{M}}$
  - The second: $\exists w' \in R(w) : \langle w', R(w)\rangle \notin [\![\mathsf{p}]\!]_c^{\mathcal{M}}$
  - These are inconsistent demands on $R(w)$!
- So epistemic contradictions are unacceptable!

- An indicative conditional semantics to predict (6) (Yalcin 2007: 998)

Ind. Conditional $[\![\phi \to \psi]\!]_c^{\mathcal{M}} = \{\langle w, R(w)\rangle \mid \forall w' \in R(w)_\phi : \langle w', R(w)_\phi\rangle \in [\![\psi]\!]_c^{\mathcal{M}}\}$

- $R(w)_\phi := \textsc{max}(s) \subseteq R(w) : (s \neq \varnothing \ \& \ \forall w' \in s : \langle w', s\rangle \in [\![\phi]\!]_c^{\mathcal{M}})$

- The basic idea is to shift the possibility sphere where ϕ has been accepted and make sure that ψ has been accepted.
  - But when ϕ is $\mathsf{p} \wedge \mathsf{Might}(\neg\mathsf{p})$, there will be no such state
- So the truth of the indicative conditional could not be defined
  - Hence epistemic contradictions are infelicitious in antecedents of indicatives
- The semantics for belief/supposition works similarly (Yalcin 2007: 995)

Belief $[\![\mathsf{Bel_A}(\phi)]\!]_c^{\mathcal{M}} = \{\langle w, R(w)\rangle \mid \forall w' \in R_A(w) : \langle w', R(w)\rangle \in [\![\phi]\!]_c^{\mathcal{M}}\}$

- $R_A(w)$ is the set of worlds compatible with what A believes in w

- It says that ϕ has been accepted in A's state of information in w
- This is false for epistemic contradictions, since no state of information accepts them!

- **Communication**: how do these new contents fit to into our account of communication?
  - Communication is, in part, the coordination on a shared body of information
  - Assertion, following Stalnaker (1970, 1978), can be thought of as adding the proposition expressed by the utterance to this body of information
    - For Stalnaker, propositions are sets of worlds
    - So, adding: intersecting two propositions
  - Intersection still works, it's just that we're intersecting sets of pairs
  - We just have a richer notion of information

- This is already implicit in modal logic: constructing an accessibility relation which makes $\Box\Box\mathsf{A}$ true and $\Box\mathsf{A}$ false requires two worlds which are 'internally identical' (they make all the same atomic claims true), but differ in their place in the accessibility relation (See notes from 01.30)

- Logical consequence can still be treated as set inclusion:
  - $\phi_1, \ldots, \phi_n \vDash \psi \iff \forall \mathcal{M}, c : [\![\phi]\!]_c^{\mathcal{M}} \cap \cdots \cap [\![\phi_n]\!]_c^{\mathcal{M}} \subseteq [\![\psi]\!]_c^{\mathcal{M}}$
- Objection (Yalcin 2007: 1011):
  - Intuitively $\neg\Diamond\phi$ follows from $\neg\phi$
  - Suppose the following. (1) Nobody – including ourselves – knows whether or not there is lead on Pluto, and indeed nobody is even close to having any evidence on the question of whether there is lead on Pluto. (2) As a matter of fact, there is no lead on Pluto. Now, on the basis of the information provided by these two premises, is the following sentence true or false?

    (7) There might be lead on Pluto

    There is a strong pull to answer 'false'.
  - Further, this calls into question the idea that epistemic modals express propositions which describe an agent's information
    - In the example we say that there is no agent with the information necessary to judge (7) false
- Yalcin says that this difficulty can be solved on an expressivist account
  - The basic idea is just like the semantics above, but we decouple the information used to evaluate modals from the world
  - s is an information state: a set of worlds
  - **Yalcin's Semantics**: $[\![\Diamond\phi]\!]_c^{\mathcal{M}} = \{\langle w, s\rangle \mid \exists w' \in s : \langle w', s\rangle \in [\![\phi]\!]_c^{\mathcal{M}}\}$
    - This handles epistemic contradictions in exactly the way discussed before
  - By decoupling s and w, epistemic modal's contents cease to convey a fact about w
    - They cease to discriminate between ways the world could be
    - All they do is in discriminate ways a state of information could be
  - So you can think of the real contribution of an epistemic modal as the set of s compatible with it: $\{s \mid \exists w' \in s : w' \in [\![\phi]\!]_c^{\mathcal{M}}\}$
    - This can't be the semantic content for compositional reasons, there's were we need the more general device of $\langle w, s\rangle$
- **Communication**: on Yalcin's view, epistemic modals do not, properly speaking, convey information, but rather properties of information
  - So how is it that they can play a role in coordinating on a body of shared information?
  - By expressing these contents, one proposes that the shared body of information come

to be among the states of information compatible with the epistemic modal

- So uttering $\Diamond\phi$ amounts to proposing that the shared body of information become one of those compatible with ϕ

- This suggests that the notion of consequence appropriate to epistemic modals is *acceptance*, not truth:
  - **Consequence** $\phi_1, \ldots, \phi_n \vDash \psi \iff \forall \mathcal{M}, c, s$ if $\phi_1, \ldots, \phi_n$ are accepted in s, so is ψ
- This predicts the inference from $\neg\phi$ to $\neg\Diamond\phi$
  - Once you've accepted $\neg\phi$ you've ruled out ϕ-worlds, so there does not exist a world that makes ϕ true.
  - That's just way $\neg\Diamond\phi$ says
- Can this consequence relation be used for a pragmatic explanation of the inference?
  - Similar to the use of reasonable inference in Stalnaker (1975)

3 Dynamic Accounts

- Think about the meaning of a computer program π
  - The computer is one state before it is run, s_1
  - Yet another after it is run, s_2
  - This led to the natural idea that π's meaning consists in the way it changes the state of the machine (Pratt 1976)
- This idea can be generalized to natural language by thinking about how a sentence ϕ changes a language user's state of mind (Heim 1982; Kamp 1981; Groenendijk & Stokhof 1991; Groenendijk *et al.* 1996; Veltman 1996)
  - Or, if you want to be a little more externalist, how it changes the object state of the conversation; the 'conversational score' (Lewis 1979)
- For basic propositional logic (s is a set of worlds, worlds are valuations):

Definition 1 (Update Semantics)

(1) $s[\mathsf{p}] = \{w \in s \mid w(\mathsf{p}) = 1\}$ (2) $s[\neg\phi] = s - s[\phi]$
(3) $s[\phi \wedge \psi] = (s[\phi])[\psi]$ (4) $s[\phi \vee \psi] = s[\phi] \cup s[\psi]$

- On this view, semantics does not assign contents or truth-conditions to formulas at all
  - It specifies their role in changing contents
- Truth is not the evaluative concept of primary importance in this semantics
- The crucial concept is **support**, and truth is a special case of it (Starr 2010)

Definition 2 (Support, Truth in w)

(1) Support $s \vDash \phi \Leftrightarrow s[\phi] = c$ (2) Truth in w $w \vDash \phi \Leftrightarrow \{w\}[\phi] = \{w\}$

- Support is essentially 'acceptance' from previous section
- von Fintel & Gillies (2007: 50) conflate these two concepts
- "The universe... would only be one fact and one great truth for whoever knew how to embrace it from a single point of view." (d'Alembert 1995: 29) [1751]
- Correspondence with the truth amounts to acceptance once uncertainty has been resolved
- So it doesn't make sense to talk about the truth conditions of claims which turn crucially on uncertainty for their use

- Since support is the key semantic notion, it is used to define logical consequence

Definition 3 (Dynamic Consequence) $\phi_1, \ldots, \phi_n \vDash \psi \Leftrightarrow \forall c : c[\phi_1] \cdots [\phi_n] \vDash \psi$

Definition 4 (Classical Consequence) $\phi_1, \ldots, \phi_n \vDash \psi \Leftrightarrow \forall w : \{w\}[\phi_1] \cdots [\phi_n] \vDash \psi$

- One can define propositions, if one wants to:

Definition 5 (Propositional Content) $[\![\phi]\!] = \{w \mid w \vDash \phi\}$

- Definitions 1 and 5 yield as corollaries the clauses of standard possible worlds semantics:

Corollary 1 (Possible Worlds Semantics)

(1) $[\![\mathsf{p}]\!] = \{w \in W \mid w(\mathsf{p}) = 1\}$ *(2)* $[\![\neg\phi]\!] = W - [\![\phi]\!]$
(3) $[\![\phi \wedge \psi]\!] = [\![\phi]\!] \cap [\![\psi]\!]$ *(4)* $[\![\phi]\!] \cup [\![\psi]\!] = [\![\phi]\!] \cup [\![\psi]\!]$

- The question then arises, couldn't one instead take these as definitions and derive Definition 1 as a corollary using Stalnaker's (1978: 86-7) pragmatic account of how assertions change context:

Stalnaker's Update Equation $s[\phi] = s \cap [\![\phi]\!]$

- The answer is *Yes*!
  - Because: we don't yet have any operators that change the context in a way that isn't identical to adding their propositional content
- But adding Veltman's (1996) semantics of *might* (which Yalcin is mimicking) changes this

Definition 6 (Dynamic Epistemic Modals)

(1) $c[\Diamond\phi] = \{w \in c \mid c[\phi] \neq \varnothing\}$ (2) $c[\Box\phi] = \{w \in c \mid c \vDash \phi\}$

- $c[\Diamond\phi] = c \cap [\![\Diamond\phi]\!]$? No.
  - Consider $\Diamond\mathsf{p}$
  - Suppose c contains one p-world, w_{p}, and many other $\neg\mathsf{p}$-worlds
  - Then, $c[\Diamond\mathsf{p}] = c$
  - What about $[\![\Diamond\mathsf{p}]\!]$? $\{w\}[\Diamond\mathsf{p}] = \{w' \in \{w\} \mid \{w'\}[\mathsf{p}] \neq \varnothing\}$
    - Or, more simply: $\{w\}[\Diamond\mathsf{p}] = \{w' \mid \{w'\}[\mathsf{p}] = \{w'\}\} = [\![\mathsf{p}]\!]$!
  - But then $c \cap [\![\Diamond\mathsf{p}]\!] = \{w_{\mathsf{p}}\} \neq c$

- Why care?
  - This account handles epistemic contradictions with ease:
    - $s[\mathsf{p} \wedge \Diamond\neg\mathsf{p}] = s[\mathsf{p}][\Diamond\neg\mathsf{p}] = \varnothing$
      - After eliminating the $\neg\mathsf{p}$-worlds the $\Diamond\neg\mathsf{p}$ test will fail!
    - This sentence is **inconsistent**: it turns any state into $\varnothing$.
    - Some sentences/discourses are **incoherent**: no non-empty state supports them
  - This notion of coherence is useful in analyzing the following difference

    (8) a. Billy might be at the door. ...It isn't Billy at the door.

    b. ?? It isn't Billy at the door. ...Billy might be at the door.
  - Can Yalcin's theory explain this kind of phenomena?

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