Kratzer on Modality in Natural Language

02.06.12

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Phil 6710 / Ling 6634 – Spring 2012

1 Modal Logic

- General Propositional modal logic:
  - **Syntax**:
    - Atomic formulas: $\mathsf{P}, \mathsf{Q}, \mathsf{R}, \ldots$, Connectives: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow, \Box, \Diamond$
      - Strict Conditional: $\Box(\phi \rightarrow \psi)$
  - **Kripke Semantics**:
    - *Model*: $\mathcal{M} = \langle\langle W, R\rangle, v\rangle$
      - Frame: $\langle W, R\rangle$
    - *Set of Possible Worlds*: W, different complete ways the world could be
    - *Accessibility*: R, a set of pairs of worlds
      - R catalogs which worlds are possible with respect to each-other
      - $R(w, w')$ means that w' is possible wrt to w
    - *Valuation*: v maps every atomic and world to a truth-value at that world, 1 or 0
    - *Truth (wrt v, w, R) Definition*:
      (1) $[\![\mathsf{A}]\!]^R_{v,w} = 1 \iff v(\mathsf{A}, w) = 1$
      (2) $[\![\neg\phi]\!]^R_{v,w} = 1 \iff [\![\phi]\!]^R_{v,w} = 0$
      (3) $[\![\phi \rightarrow \psi]\!]^R_{v,w} = 1 \iff [\![\phi]\!]^R_{v,w} = 0$ or $[\![\psi]\!]^R_{v,w} = 1$
      (4) $[\![\phi \wedge \psi]\!]^R_{v,w} = 1 \iff [\![\phi]\!]^R_{v,w} = [\![\psi]\!]^R_{v,w} = 1$
      (5) $[\![\Box\phi]\!]^R_{v,w} = 1 \iff$ for every w' $R(w, w') : [\![\phi]\!]^R_{v,w'} = 1$
      (6) $[\![\Diamond\phi]\!]^R_{v,w} = 1 \iff$ for some w' $R(w, w') : [\![\phi]\!]^R_{v,w'} = 1$
      $\vdots$
  - **Consequence**:
    - $\phi_1, \ldots, \phi_n \vDash \psi \iff$ For every $\mathcal{M}, w$: $[\![\psi]\!]^R_{v,w} = 1$ if $[\![\phi_1]\!]^R_{v,w} = \cdots = [\![\phi_n]\!]^R_{v,w} = 1$
      - $\mathcal{M} = \langle\langle R, W\rangle, v\rangle$ and $w \in W$

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- Particular propositional modal logics:
  - When defining what a model is:
    - State requirements on R
      - E.g. reflexivity
  - This limits the models considered when evaluating consequence
  - As a result: certain inferences are predicted as valid
    - E.g. reflexivity guarantees: $\Box\phi \vDash \phi$
- Philosophical question:
  - Do facts about accessibility *explain* why some sentences follow from others?

2 Modality in Natural Language

2.1 Kinds of Modality

- In this class, our terminology:

Epistemic Relates p to information (evidence, belief, knowledge, documents, etc.)

(1) "When you have eliminated the impossible, whatever remains, however improbable, must be the truth." (Sherlock Holmes in *The Sign of Four*)

(2) Gauging by the footprints, the bear must weigh 300lbs.

(3) According to police records, the criminal might be from Arizona.

(4) "One eyewitness a former royal guard called Druk Sherrik described his encounter with the... yeti. 'It was huge. It must have been nine feet tall.' "

Deontic Relates p to expectations (laws, desires, ideals, etc.)

(5) "Ideally, a prosthesis must be comfortable to wear, easy to put on and remove, light weight, durable, and cosmetically pleasing."

(6) "Oxfam: To fully observe law, Obama's fight against LRA must focus on civilian protection."

(7) "According to the *Bible*, human life must be respected and protected absolutely from the moment of conception."

Root Relates p to dispositions of things

(8) Light must travel at 299,792,458 m/s

(9) "The finite speed of light also limits the theoretical maximum speed of computers, since information must be sent within the computer from chip to chip."

(10) A flower can grow here

- For a roadmap of how this terminology is actually employed by linguists and a different proposal see Portner (2009: Ch.4)

2.2 Kinds of Modality, Meet Modal Logic

(11) According to his dating coach, John must dance at parties. (Deontic)

- Does not entail: John dances at parties

(12) Since John hangs out with Linda at parties, he must dance at parties. (Epistemic)

- Does entail: John dances at parties

- How can we give a semantics for $\Box$ that captures both facts?
- A problem for modal logic analysis:
  - To predict lack of inference in (11) ($\mathsf{Must}\,\phi \nvDash \phi$) R can't be reflexive
  - To predict inference in (12) ($\mathsf{Must}\,\phi \vDash \phi$) R must be reflexive
- One solution:
  - Allow a frame to have **multiple accessibility relations**:
    - $\mathcal{M} = \langle\langle W, R_d, R_e\rangle, v\rangle$
    - Require R_e to be reflexive, but not R_d
  - Define multiple Must operators: $\mathsf{Must}_d, \mathsf{Must}_e$
    - $[\![\mathsf{Must}_d\,\phi]\!]^{R_d,R_e}_{v,w} = 1 \iff$ for every w' $R_d(w,w')$: $[\![\phi]\!]^{R_d,R_e}_{v,w'} = 1$
    - $[\![\mathsf{Must}_e\,\phi]\!]^{R_d,R_e}_{v,w} = 1 \iff$ for every w' $R_e(w,w')$: $[\![\phi]\!]^{R_d,R_e}_{v,w'} = 1$
- This handles nicely the fact that deontics and epistemics mix:
  - *John must donate to charity, and he might do so*: $\mathsf{Must}_d\,\mathsf{C} \wedge \mathsf{Might}_e\,\mathsf{C}$
  - $[\![\mathsf{Must}_d\,\mathsf{C} \wedge \mathsf{Might}_e\,\mathsf{C}]\!]^{R_d,R_e}_{v,w} = 1 \iff [\![\mathsf{Must}_d\,\mathsf{C}]\!]^{R_d,R_e}_{v,w} = [\![\mathsf{Might}_e\,\mathsf{C}]\!]^{R_d,R_e}_{v,w} = 1$
- Problems w/this approach (Kratzer 1977):
  - Surely there's only one word *must*! Not *must*$_d$ and *must*$_e$
  - Whether or not you get deontic/epistemic interpretation depends on surrounding linguistic context
    - E.g. on the modal logic analysis predicts deontic reading of (2) to be equally natural
    - But it's due to *Gauging by the footprints...*
  - How do we account for the indefinite variety of meanings among the deontic uses?
    - Even if *must* in (5)-(7) has the same logic, *must p* seems to communicate something different in each case
    - A different ordering for all of the possible restrictions is obviously a hack

- Another problem: **graded modality**
  - Consider these:

    (13) Chaz is probably the murderer

    (14) There is a good possibility that Chaz is the murderer

    (15) There is a slight possibility that Chaz is the murderer

    (16) It is more likely that Chaz is the murder than Alf
  - Maybe we can account for (13) by talking about *most* accessible worlds
  - But how to capture (14)-(16) by quantifying over the accessible worlds?
- Yet another problem: **inconsistency**
  - w_0: judges set absolutely the laws with their judgements
    - Judge A:
      - No murder
      - Owners of goats are pay for damage their animals inflict on flowers and vegetables
    - Judge B:
      - No murder
      - Owners of goats **don't** pay for damage their animals inflict on flowers and vegetables
  - Intuitions about the laws in w_0:
    - True:

      (17) You must not murder

      (18) Owners of goats may have to pay for damage they cause

      (19) Owners of goats may not have to pay for damage they cause
    - False:

      (20) You must murder
  - $R(w_0, w)$ iff all of the judges judgements in w_0 are true in w
    - Oops, there aren't any such worlds!
    - That makes (20) true
    - And both (18) and (19) false

3 Kratzer's Theory of Modality

- Minor shift in sentence meanings
- Instead of denoting a truth-value at a world, sentences denote a set of worlds
  - The set of worlds where the sentence is true

  Informational content can be understood in terms of possibilities. The information admits some possibilities and excludes others. Its content is given by the division of possibilities into the admitted ones and the excluded ones. The information is that some one of *these* possibilities is realized, not any of *those*. (Lewis 1983: 4)

- We will call these sentence meanings **propositions**
- For example, old modal logic semantics:

(1) $[\![\mathsf{A}]\!]^R_{v,w} = 1 \iff v(\mathsf{A}, w) = 1$

(2) $[\![\neg\phi]\!]^R_{v,w} = 1 \iff [\![\phi]\!]^R_{v,w} = 0$

(3) $[\![\phi \wedge \psi]\!]^R_{v,w} = 1 \iff [\![\phi]\!]^R_{v,w} = [\![\psi]\!]^R_{v,w} = 1$

(4) $[\![\Box\phi]\!]^R_{v,w} = 1 \iff$ for every w' $R(w,w') : [\![\phi]\!]^R_{v,w'} = 1$

(5) $[\![\Diamond\phi]\!]^R_{v,w} = 1 \iff$ for some w' $R(w,w') : [\![\phi]\!]^R_{v,w'} = 1$

- Compare with new modal logic semantics:

(1) $[\![\mathsf{A}]\!]^R_{v} = \{w \mid v(\mathsf{A}, w) = 1\}$

(2) $[\![\neg\phi]\!]^R_{v} = W - [\![\phi]\!]^R_{v}$

(3) $[\![\phi \wedge \psi]\!]^R_{v} = [\![\phi]\!]^R_{v} \cap [\![\psi]\!]^R_{v}$

(4) $[\![\Box\phi]\!]^R_{v} = \{w \mid \{w' \mid R(w,w')\} \subseteq [\![\phi]\!]^R_{v}\}$

Truth $\mathcal{M}, w \vDash \phi \iff w \in [\![\phi]\!]^R_v$

Consequence $\phi_1, \ldots \phi_n \vDash \psi \iff \forall \mathcal{M}, w : ([\![\phi_1]\!]^R_v \cap \cdots \cap [\![\phi_n]\!]^R_v) \subseteq [\![\psi]\!]^R_v$

Consistency $A = \{p_1, \ldots, p_n\}$ is consistent iff $\bigcap A \neq \varnothing$

- Note: $\bigcap A := \{w \mid \forall p \in A : w \in p\}$
  - E.g. $\bigcap A = p_1 \cap \cdots \cap p_n$
  - And: $\bigcap \varnothing = W$

Compatibility p is compatible with A iff $A \cup \{p\}$ is consistent, $(\bigcap(A \cup \{p\}) \neq \varnothing)$

Conversational Background The thing denoted by phrases like *what the law provides*. The law provides a set of propositions that need to be made true. But just which propositions varies from world to world. So a conversational background can be modeled as a function from worlds to sets of propositions.

- E.g. $f(w_1) = \{p_1, p_2\}, f(w_2) = \{p_2, p_3\}$
- Think of $\bigcap f(w)$ as the conjunction of all the propositions that have to be true in w

3.1 Step 1: Relative Modality

- Each modal sentence has three parts:
  - Conversational background
  - Modal particle
  - Sentence over which modal scopes
- Their form: $\mathsf{CB}\,(\mathsf{Modal}\,\phi)$
- Their semantics:
  - $[\![\mathsf{CB}\,(\mathsf{Modal}\,\phi)]\!]^f_v = [\![\mathsf{Modal}\,\phi]\!]^{[\![\mathsf{CB}]\!]}_v$
  - CB sets the conversational background wrt which the modal is interpreted!
- The modal semantics:
  - $[\![\mathsf{Must}\,\phi]\!]^f_v = \{w \mid \bigcap f(w) \subseteq [\![\phi]\!]^f_v\}$
    - ▶ The conversational background entails ϕ
  - $[\![\mathsf{May}\,\phi]\!]^f_v = \{w \mid (\bigcap f(w)) \cap [\![\phi]\!]^f_v \neq \varnothing\}$
    - ▶ The conversational background is compatible with ϕ
- This point:
  - You get all the flavors and sub-flavors of modal meanings out of one lexical entry
  - When CB doesn't explicitly occur in the sentence, there is a 'silent' or implicit one
- Other than the idea of how modals are relativized to conversational backgrounds, this semantics is equivalent to modal logic
  - Define R in terms of f: $R(w,w') \iff w' \in \bigcap f(w)$
- Outstanding problems:
  - Graded modality
  - Inconsistency

3.2 Step 2: Ordering Semantics Modality

Essential Innovation There are two conversational backgrounds relevant to each modal: the **modal base** and the **ordering source**

a. The modal base is f: it determines which worlds are accessible

b. The ordering source is g: it is used to induce an ordering on the accessible worlds

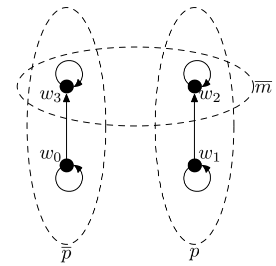
- How does g order a set of worlds?
  - $w' \leq_{g(w)} w'' \iff \{p \in g(w) \mid w'' \in p\} \subseteq \{p \in g(w) \mid w' \in p\}$
    - ▶ Every proposition from $g(w)$ which w'' makes true, w' also makes true

- ▶ $\leq$ reverses $\subseteq$
- ▶ According to the ideal $g(w)$: w' is at least as good as w''
- ○ Following Lewis (1981: 220), Kratzer (1991: 644) says that $\leq_{g(w)}$ is a **partial order**
- ○ Normally, a partial order is defined as follows:
  - ▶ Reflexive: for all w', $w' \leq_{g(w)} w'$
  - ▶ Antisymmetric: $w' = w''$ if $w' \leq_{g(w)} w''$ and $w'' \leq_{g(w)} w'$
  - ▶ Transitive: $w' \leq_{g(w)} w'''$ if $w' \leq_{g(w)} w''$ and $w'' \leq_{g(w)} w'''$
- ○ But $\leq_{g(w)}$ isn't antisymmetric!
  - ▶ Set $g(w) = \{\{w_0, w_1\}$
  - ▶ Every proposition w_0 makes true, w_1 makes true
    - ▷ So $w_1 \leq_{g(w)} w_0$
    - ▷ Remember: $\leq$ reverses $\subseteq$
  - ▶ Every proposition w_1 makes true, w_0 makes true
    - ▷ So $w_0 \leq_{g(w)} w_1$
  - ▶ Yet $w_1 \neq w_0$!
- ○ Hence $\leq$ is merely a **preorder** (reflexive, transitive)
  - ▶ Lewis (1981) must have been using *partial order* in an idiosyncratic way, because he explicitly acknowledges that $\leq$ admits of 'ties': non-identical w, w' where $w \leq w'$ and $w' \leq w$.

- New modal semantics:
  - ○ $[\![\textsf{Must}\,\phi]\!]_v^{f,g} = \{w \mid \textsc{Best}(f(w), g(w)) \subseteq [\![\phi]\!]_v^{f,g}\}$
    - ▶ $\textsc{Best}(f(w), g(w)) := \{w' \in \bigcap f(w) \mid \nexists w'' \neq w' \in \bigcap f(w) : w'' \leq_{g(w)} w'\}$
  - ○ $[\![\textsf{May}\,\phi]\!]_v^{f,g} = \{w \mid \textsc{Best}(f(w), g(w)) \cap [\![\phi]\!]_v^{f,g} \neq \varnothing\}$
  - ○ This makes the **limit assumption**
    - ▶ Worlds don't get better and better off to infinity
      - ▷ Example: Sally's happiness corresponds to the real number she's currently thinking of
    - ▶ Relaxing the assumption makes for a more complex definition of necessity
  - ○ The limit assumption doesn't matter for Kratzer's account of inconsistency
  - ○ So after using this simple version to see how her account of inconsistency works, we'll examine the more complex definition.

3.2.1 Inconsistency

- $f(w_0) = \varnothing$, so $\bigcap f(w_0) = W$
  - ○ Kratzer: let's just make no assumptions about which worlds are accessible
  - ○ For our purposes, let's assume: $W = \{w_0, w_1, w_2, w_3\}$
- $g(w_0) = \{\overline{m}, p, \overline{p}\}$, where $\overline{p}$: $W - p$
  - ○ m: set of worlds where murder happens $\{w_0, w_1\}$
  - ○ p: set of worlds where everyone pays for goat-damage $\{w_1, w_2\}$
  - ○ Reading ${}_{w}\bullet\!\!\longrightarrow\!\!\bullet_{w'}$ as $w' \leq w$ (w' is at least as good as w):



- Calculating $\textsc{Best}(f(w_0), g(w_0))$:
  - ○ Construct $\leq_{g(w_0)}$
    - ▶ The set propositions w_0 makes true: $\{\overline{p}\}$
    - ▶ Set of propositions w_1 makes true: $\{p\}$
    - ▶ Set of propositions w_2 makes true: $\{\overline{m}, p\}$
    - ▶ Set of propositions w_3 makes true: $\{\overline{m}, \overline{p}\}$
    - ▶ Which of these stand in the subset relation? ($\leq$ is reverse $\subseteq$)
      - ▷ So $w \leq w'$ if the propositions w' makes true is a subset of the propositions w makes true
    - ▶ $w_0 \leq_{g(w_0)} w_0$ since $\{\overline{p}\} \subseteq \{\overline{p}\}$
      - ▷ Same reasoning holds for all worlds
    - ▶ $w_3 \leq_{g(w_0)} w_0$, $w_2 \leq_{g(w_0)} w_1$
    - ▶ No other subset relations obtain, so all the other worlds are incomparable
  - ○ So $\textsc{Best}(f(w_0), g(w_0)) = \{w_2, w_3\}$
    - ▶ Does not entail $[\![\textsf{Murder}]\!]_v^{f,g}$, so (17) is true and (20) is false
    - ▶ Compatible with $[\![\textsf{Pay}]\!]_v^{f,g}$, so (18) is true
    - ▶ Compatible with $[\![\neg\textsf{Pay}]\!]_v^{f,g}$, so (19) is true

3.2.2 Graded Modality

Let $u, v, z \in \bigcap f(w)$ (Kratzer 1991: §3.3):

Necessity of p in w
$\forall u\, \exists v : v \leq_{g(w)} u \;\&\; \forall z : z \in p$ if $z \leq_{g(w)} v$

- For every world u, there is a world v such that both:
  - v is at least as good as u
  - Every world at least as good as v makes p true

Possibility of p in w
$\exists u\, \forall v : v \not\leq_{g(w)} u$ or $\exists z : z \leq_{g(w)} v \;\&\; z \in p$

- There is a world u, such that for every world v, either:
  - v is not at least as good as u
  - There is a world at least as good as v which makes p true

Good Possibility of p in w
$\exists u\, \forall v : v \in p$ if $v \leq_{g(w)} u$

- There is a world such that every world at least as good as it makes p true

At Least As Good Possibility of p as q in w
$\forall u \in q\; \exists v : v \leq_{g(w)} u \;\&\; v \in p$

- For every q-world, there is a p-world which is at least as good as it.

Better Possibility of p than q in w
$\forall u \in q\; \exists v : v \leq_{g(w)} u \;\&\; v \in p$, and not $\forall u \in p\; \exists v : v \leq_{g(w)} u \;\&\; v \in q$

- p is at least as good of a possibility as q, but not vice-versa

Weak Necessity of p in w
$\forall u \in \overline{p}\; \exists v : v \leq_{g(w)} u \;\&\; v \in p$

- p is a better possibility than $\overline{p}$
- E.g. *probably*

Slight Possibility of p in w
$\exists u\, \forall v : v \not\leq_{g(w)} u$ or $\exists z : z \leq_{g(w)} v \;\&\; z \in p$, and $\forall u \in p\; \exists v : v \leq_{g(w)} u \;\&\; v \in \overline{p}$

- p is a possibility, but $\overline{p}$ is a weak necessity (probable)

3.2.3 Modal Typology

	modal force	modal base	ordering source
muss	necessity	no restrictions	no restrictions
kann	possibility	no restrictions	no restrictions
darf	possibility	circumstantial	deontic, teleological ('in view of certain aims')
soll$_1$	necessity	circumstantial	bouletic ('in view of certain wishes')
soll$_2$	necessity	empty	hearsay
wird	weak necessity	epistemic	doxastic ('in view of certain beliefs')
dürfte	weak necessity	epistemic	stereotypical

3.3 Conditionals: *if* Restricts Modal Base

3.4 Defining Consequence

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