Conditionals and Questions

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1 Introduction

Conditional sentences, such as (1) and (2), are a heavily worked resource in the activities of planning, communication and inquiry.

(1) If Bob danced, Leland danced ('Indicative')

(2) If Bob had danced, Leland would have danced ('Subjunctive')

Their study has unearthed phenomena which have dramatically influenced semantic theory and views on its role in the explanation of these activities. Frege (1893), Jeffrey (1963), Grice (1989a) and others, used the tools of **truth-functional semantics**. They model the meaning of *if* as a binary truth-function that computes the truth-value of the conditional from the truth-values of the antecedent and consequent. C.I. Lewis (1914), Stalnaker (1968), D.K. Lewis (1973) and others explore a **possible-worlds semantics**. They render *if* as a binary propositional function, taking two sets of possible worlds (propositions) to a third one, the conditional proposition.[1] These truth-conditional

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[1] To simplify matters, I will suppress discussion of Kratzer's (1986; 1991) **restrictor theory** which differs in compositional detail from connective theories. This theory also constructs conditional propositions using possible-worlds semantics, but maintains that conditionals are held together by a (often covert) binary modal operator and relegates *if* to a supporting role: semantic vacuity or serving only to restrict the modal. This approach is equally frustrated by (3)-(5). Here, *if* neither syntactically restricts a binary modal nor wilts into semantic vacuity.

connective theories are canonically distinguished from **suppositional theories** (e.g. Quine 1950: 21; von Wright 1957: 131; Mackie 1973: Ch.4; Adams 1975: 1-42; Edgington 1995: §§7-9) which maintain that the acceptance or assertion of a conditional does not involve the acceptance or assertion of a conditional proposition. Instead, the *if*-clause contributes a supposition under which the consequent alone is accepted or asserted. There is ambivalence about the theory's semantic foundations. But all variants endorse a departure from the truth-conditional model and most adopt a **probabilistic semantics**.[2]

Much recent debate has focused on which of these two approaches should be adopted (e.g. Lycan 2006; Edgington 2008) and is a rare case where truth-theoretic and use-theoretic perspectives compete and engage. The exclusive advertised benefits of suppositional theories are their treatment of conditionals' probabilities (Adams 1975; Bennett 2003; Edgington 2008) and indicative conditionals' sensitivity to the private epistemic states of language-users (Gibbard 1981; Edgington 1995; Bennett 2003). Truth-conditional theories emphasize their ease in explaining truth-value judgements, adhering to compositionality, treating indicative and subjunctive conditionals uniformly and integrating with truth-conditional frameworks used for other regions of discourse.

There is, however, one phenomenon that neither approach can accommodate, namely non-conditional, interrogative occurrences of *if* (Harman 1979: 48).[3]

(3) Albert wondered if Mabel loved John
(4) Mabel asked if John was going to the party

To these specimens I add (5).

(5) The future is coming. The question is if we will be ready for it.

Here we find an isolated *if*-clause introducing a question as the *argument* of an interrogative attitude verb or the identity relation. There is simply no supposition and no binary operation on propositions or truth-values.

[2] E.g. Adams (1975); Appiah (1985); McGee (1989); Edgington (1995); Bennett (2003). Belnap (1973) hybrids connective and suppositional accounts by, essentially, using a three-valued logic. There are numerous, more tenebrous, versions.
[3] See also Haiman (1978).

The convergence of interrogatives and conditional antecedents is very common even across *unrelated* languages,[4] a pattern which makes lexical ambiguity approaches both implausible and unexplanatory. This **conditional-interrogative link** should not be construed as an identification of *all* conditional antecedents in all languages with interrogatives, and need not be for it to be of theoretical importance.[5] Their widespread overlap requires explaining how it is that a language could use the same morpheme to form a conditional antecedent and an embedded interrogative. Whatever the abstract semantic structure of conditionals is, it must be flexible enough to frame an answer to this question and hence must not be what existing theories take it to be.

This paper argues that accommodating the conditional-interrogative link does not merely capture a phenomena that eludes the two dominant approaches. It leads to a new perspective on the debate between them and directly impacts the issues that make that debate important. Like those approaches, the semantics proposed in §2.5 has important implications for the shape of semantic theory, but it also shows that one need not choose between the perspectives and benefits of those two approaches. The meaning of a sentence is identified with the characteristic role it plays in changing mental states of language users. Truth-conditions are determined by this role, but I explore a view of entailment and language use from the more general perspective (§2.3). This allows for a model of communication whereby agents can convey features of their epistemic situation without directly reporting it (§2.5). It also allows me to combine a successful logic of natural language indicative conditionals (§3.1) with a desirable account of their truth-conditions (§3.2) in a way no other semantics has. The semantics offered in §2.5 can be extended to subjunctive conditionals to deliver a uniform account of the two varieties, but that will have to wait for another occasion [reference suppressed].

[4] As documented in Kayne (1991: §2.2), French *si* and Italian *se* occur both in conditionals and under interrogative verbs; the same pattern holds in Spanish. Similarly for Bulgarian and many of the Slavic languages (Bhatt & Pancheva 2006: 653). The pattern is also prominent in non-Indo-European languages, occurring in Hebrew (Roger Schwarzschild p.c.), Hua, Mayan Tzotzil and Tagalog (Haiman 1978: 570), just to name a few. In American Sign-Language (ASL) and Italian Sign-Language (LIS) the same non-manual articulation marks the antecedents of conditionals and interrogatives: a raised brow (Pyers & Emmorey 2008, Adriana Belletti p.c.).

[5] Even in English there are sentences with conditional meanings but no plausibly interrogative antecedent, e.g. *q provided that p*, *q given that p*.

2 A New Semantics for Conditionals

As Austin (1956: 211-2) reminds us:

> The dictionary tells us that the words from which our *if* is descended expressed, or even meant, 'doubt' or 'hesitation' or 'condition' or 'stipulation'. Of these, 'condition' has been given a prodigious innings by grammarians, lexicographers, and philosophers alike: it is time for 'doubt' and 'hesitation' to be remembered...

Considering several paraphrases of *I can if I choose* he observes:

> ...[W]hat is common to them all is simply that the *assertion*, positive and complete, that 'I can', is linked to the *raising of a question* whether I choose to, which may be relevant in a variety of ways. (Austin 1956: 212; original emphasis)

This passage is intended as a remark on one sense of *if*. However, I shall contend that it provides a general insight about conditionals: *q if p* links the assertion of *q* to the raising of a question *p?* This insight provides the key to understanding the conditional-interrogative link.

2.1 First Steps

Begin with the interrogative side of the link, considering occurrences of *if* like (3) and (4) above. The leading hypothesis about their semantics relies on the leading hypothesis about the semantics of interrogatives due to Hamblin (1958).[6] Hamblin's central idea was that the meaning of an interrogative is not given by its truth-conditions, but rather by its answerhood-conditions. A **polar** (yes/no) interrogative like (6a) has two complete and direct answers: (6b) and (6c).[7] It thus presents two exclusive and exhaustive alternative propositions. An answer to it consists in selecting exactly one of them. Accordingly, (6a)'s answerhood-conditions can be identified with the set containing these two propositions, i.e. Q_b in (7). On analogy with the terminology of *propositions*, this set is often called a *question* (Higginbotham 1996: 362).

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[6] Force-based approaches (e.g. Searle 1969) are inferior in numerous ways. See Higginbotham (1993: §6), Groenendijk & Stokhof (1997: §§3.3-3.6) and Belnap (1990: §§1-2).

[7] This extends to interrogatives like *Who danced?*, but these aren't relevant here.

(6) a. Did Bob dance?
b. Yes, Bob danced
c. No, Bob didn't dance

(7) $Q_b = \{b, \bar{b}\}$
$b =$ the proposition that Bob danced
$\bar{b} =$ the proposition that Bob didn't dance

Believes in (8a) expresses a relation between Cooper and the proposition denoted by *that Bob danced*. Similarly, *wonder* in (8b) expresses a relation between Cooper and the question denoted by *if Bob danced*.

(8) a. Cooper believes that Bob danced
b. Cooper wonders if Bob danced

The conditional-interrogative link compels us to wonder how this question could combine with the meaning of *Leland danced* to yield a plausible meaning for *if Bob danced, Leland danced*. Austin hints that they could be, but how?

The following discourses provide counsel, their genre inspiring the label **advertising conditional**.

(9) Do you need an efficient car? (Then) Honda has the vehicle for you

(10) Single? You haven't visited Match.com

(11) Art thou bound unto a wife? Seek not to be loosed. Art thou loosed from a wife? Seek not a wife.
(*Corinthians 7:27*, cited by Jespersen 1940: 374)

Jespersen (1940: 374) proposes that the conditional interpretations in (11) arise from each command being issued against a background where an affirmative answer (*yes*) to its preceding question is *supposed*. Each sequence thereby comes to have a conditional meaning, just as *supposing p, q!* does. With modifications, this idea provides an account of the 'link' between the consequent and interrogative antecedent of a conditional *sentence*.[8] This account begins

[8] This extension to the sentential domain is foreshadowed by German, among other languages, in its parallel use of word-order to identify the antecedent of a conditional and an interrogative.

(12) Hast du was, dann bist du was
Have you something, then are you something

with a certain characterization of the relationship between conditional sentences and suppositional reasoning.

F.P. Ramsey's enduring remark draws together conditionals and supposition:

> If two people are arguing 'If *p*, will *q*?' and are both in doubt as to *p*, they are adding *p* hypothetically to their stock of knowledge, and arguing on that basis about *q*. . . (Ramsey 1931: 247)

On this view, evaluating a conditional involves a *hypothetical* addition to the information being taken for granted, which is precisely what supposition involves. Ramsey notes a connection between this process and doubting if *p* (see also Grice 1989a: 75-8), but makes little of it. Inquiry and communication not only take place against a background of information but also a background of issues. These issues are questions left open by the background information. But, more importantly, they are questions that have been distinguished as ones that the agents are out to settle. On Hamblin's picture, these questions are a cluster of epistemically open, exhaustive and incompatible propositions the agents are aspiring to decide between. This richer picture of inquiry and communication brings one closer to making sense of the interrogative antecedents of conditionals. To see this, enrich Ramsey's remark in the following way: if two people are arguing 'if *p*, will *q*?', they are hypothetically adding *p?* to their stock of issues, then further hypothesizing a *yes*-resolution of that issue (a lá Jespersen) and arguing on that basis about *q* (thereby linking the assertion of *q* to the raising of a question *p?* a lá Austin). If the sole contribution of *if p* to this process is the addition of *p?*, then the proposal is on track to accommodate the conditional-interrogative link. The process can be so-rendered, but I shall return to this task after elaborating the proposal in more detail.

According to the proposal above, evaluating a conditional *q if p* consists in (i) hypothetically taking an interest in deciding between *p* and *not-p*, i.e. the question *p?*, (ii) further hypothesizing a *p* outcome and (iii) concluding that *q* follows from this outcome.[9] Exploiting the parallel between supposition and

'If you have something, then you are something'

(Bhatt & Pancheva 2006: 644; see also Embick & Iatridou 1994)

[9] It might seem arbitrary to require that a *positive* answer to *if p* is hypothetically adopted. At the end of §2.5 I will motivate this assumption.

'hypothetical additions', this proposal can be clarified by providing a *rough* paraphrase of a conditional sentence like (1) in terms of a suppositional discourse like (1′).

(1) If Bob danced, Leland danced

(1′) a. Suppose that we are wondering if Bob danced...

b. ...and it turns out he did.

c. Then it will follow that Leland danced.

This method of interpreting conditionals captures their core semantic property, namely **modus ponens**: *if* p *then* q and p entails q.[10] Interpreting a conditional is positioning oneself to apply modus ponens. This involves taking the consequent to follow from the antecedent. But it also involves entertaining the question *p?* This in turn requires clearly distinguishing live p and *not-p* possibilities, and taking an interest in finding out which class the actual world belongs to. The richer picture construes conditionals as a more complete microcosm of inquiry: they involve entertaining an issue and exploring the consequences of a certain hypothesis about its resolution.

I have described the meaning of a conditional in terms of a *process* but I am looking for a *semantics*. Have I found it? It is universally accepted that models of how language-users track an unfolding process play a key role in explaining how they use language to get things done. My claim is that explicitly encoding this process in the semantics of sentences gives a more general, more streamlined and more perspicuous account of how *if* fits into the grammar of English. Above, that process was specified as a transition from one 'body of information and issues' to another, one that involved 'hypothetical additions'. This proposal will be developed in three phases. I will begin by adopting a model of the bodies of information and issues in question (§2.2) and then introduce the basic ideas of a semantics based on transitions between them (§2.3). I will then offer a model of hypothetical additions to these bodies of information and issues (§2.4). The resources of these three sections are combined in §2.5 to provide a semantics for conditionals according to which the role of *if* in conditionals and embedded interrogatives is the same. Though I will be concerned here exclusively with indicative conditionals, the approach is extended to subjunctives in [reference suppressed].

[10] The theory in §2.5 provides a compelling diagnosis of McGee's (1985) attempts to counterexample modus ponens. This diagnosis is proposed by Gillies (2004: §3).

2.2 From Information to Issues

What is information? I won't hazard an answer to that difficult question here. I will merely adopt a standard and convenient model of information to lend precision to my discussion of the questions I am out to answer. The model:

> Informational content can be understood in terms of possibilities. The information admits some possibilities and excludes others. Its content is given by the division of possibilities into the admitted ones and the excluded ones. The information is that some one of *these* possibilities is realized, not any of *those*. (Lewis 1983: 4)

Formally, an informational content (*proposition*) can be identified with a set of possible worlds (Stalnaker 1976). This set distinguishes ways the world might be (worlds in the set) from ways it isn't (worlds excluded from the set).

This model also lends precision to Hamblin's picture of interrogative content. His picture was that the content of a polar interrogative is identified with the set $\{p, \overline{p}\}$. On the present model, this set amounts to a division of the space of possibilities into two mutually exclusive and exhaustive sets called **partitions**. Figures 1 and 4 illustrate these ideas about content for an impoverished space of worlds containing just one two-dimensional object a, which can have four different shapes. Let p be the proposition that 'a is a polygon'. To understand

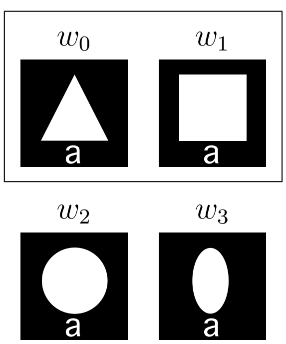

Fig. 1. The proposition p

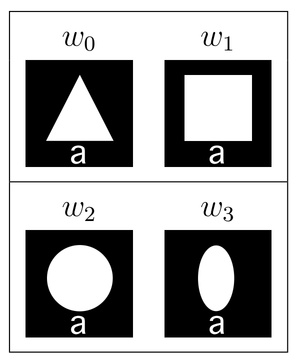

Fig. 2. The question $\{p, \overline{p}\}$

communication and inquiry, it is necessary to consider the body of information that accumulates as the process unfolds. Think of this information as what the agents are taking for granted in some way. Call the set of worlds encoding this information c, short for **contextual possibilities**. Grice, Lewis, Stalnaker and others view this background information as what's *mutually* taken for granted.[11] I make no specific assumptions about the attitude that defines c, though §3.3 will returns to that topic.

To understand communication and inquiry, it necessary to consider more than just the information that gets taken for granted. Recent work in epistemology, semantics and pragmatics makes clear that representing the issues at stake in that activity is also crucial.[12] This can be achieved by dividing c into partitions. These partitions represent open, incompatible propositions the agents are concerned with deciding between; their **issues**.[13] Formally, this division of c may be identified with a set of sets of worlds which I will refer to as C. So if there are no issues and $c = p$ from Fig. 1, the acceptance of the question of whether a is a square involves a transition from $\{\{w_0, w_1\}\}$ to $\{\{w_0\}, \{w_1\}\}$. Given their relationship, c can always be constructed from C by merging the members of C, i.e. $\bigcup C = c$. This allows one to state all changes in terms of changes to C. New information eliminates worlds from the members of C. New issues further divide the members of C. See Appendix A.2 for details.

2.3 Semantics, Linguistic Meaning and Logic

On the standard approach to semantics sentences are paired with contents.[14] A declarative sentence p is paired with an informational content $[\![\mathsf{p}]\!]$ and an interrogative sentence $?\mathsf{p}$ is paired with a question $[\![?\mathsf{p}]\!]$. The process by which these contents are incorporated into C is held to be a matter of pragmatics. That is, the process is supposed to be regulated by general principles of rational coordination, not linguistic competence.

The kind of semantics sketched in §2.1 was different. There, the linguistic meaning of an expression was a transition from one 'body of information and issues' to another, i.e. a transition from one content to another. It thereby redraws the relationship between content and linguistic meaning, and the role linguistic competence plays in changing C. The goal of this section is to give

[11] E.g. Lewis (1969, 1979), Grice (1989b) and Stalnaker (1999, 2002). For more on the representation of this attitude see Fagin *et al.* (1995); Clark (1996: Ch.4).
[12] E.g. Jeffrey (1990: Ch.11), Schaffer (2004), Groenendijk (1999), Roberts (2004) and Yalcin (2008).
[13] *Open* since they are composed from worlds in c. *Incompatible* since they are disjoint. Objects of concern, since they are the alternatives in c.
[14] Or, when context-sensitivity lurks, functions from contexts to contents.

a basic sketch of a semantics with this format and make these points more explicit. Towards this end, I will begin by specifying simpler transitions in terms of sets of contextual possibilities rather than C.

A semantics stated in terms of transitions from one informational content to another can be modeled by letting the semantic value of ϕ be a function $[\phi]$ that maps one set of possibilities to another, writing $c[\phi] = c'$ to mean that c' is the result of applying $[\phi]$ to c.[15] Read $c[\phi] = c'$ as: c' is the result of updating c with ϕ.[16] This equation identifies a sentence's meaning with its **information change potential** (ICP).[17] An ICP is just a way of modifying a set of possibilities, changing the information it embodies. The content of c is defined by whatever acceptance attitude is appropriate to modeling communication and inquiry. So, to say that a sentence ϕ of a speaker S's language has a given ICP is just to say that ϕ plays a characteristic role in changing some of S's mental states, a role specified in terms of how the contents of those states change. These characteristic changes may come in the wake of speech acts, where ϕ changes the content of the attitude defining c, and thoughts where ϕ may change the content of less public attitudes.[18] How does this **dynamic** approach relate to truth-conditional ones? This will become clear below where I consider a specific example of such a semantics.

Consider a propositional language with the familiar syntax, starting with a set of atomic sentences $\mathcal{A}t = \{\mathsf{p}_0, \mathsf{p}_1, \ldots\}$. A possible world will be treated as an assignment of one truth-value, either 1 (True) or 0 (False), to every atomic sentence. The meanings of sentences are specified in the format discussed above.

Definition 1 (Update Semantics)

(1)	$c[\mathsf{p}] = \{w \in c \mid w(\mathsf{p}) = 1\}$	(2)	$c[\neg\phi] = c - c[\phi]$
(3)	$c[\phi \wedge \psi] = (c[\phi])[\psi]$	(4)	$c[\Diamond\phi] = \{w \in c \mid c[\phi] \neq \varnothing\}$

[15] In more familiar notation: $[\phi](c) = c'$.

[16] The terminology of 'update' and the general format of this semantics originates with Veltman (1996) (circulated in 1990) and it is closely related to Heim's (1982) system. Pratt (1976) is the earliest precursor I am aware of.

[17] To pay homage to Heim's (1982) terminology of *context change potentials*.

[18] I will not be able to address the interesting question of whether the communicative role (Grice 1957), the cogitative role (Harman 1975; Chomsky 1964: 58-9) or neither role is *constitutive* of a sentence's linguistic meaning.

(1)-(4) assign each kind of formula a special role in modifying c. Atomic sentences eliminate possibilities incompatible with their truth. Conjunctions update with each of their conjuncts in sequence. Negation eliminates the possibilities compatible with its scope. (4) approximates epistemic *might* (Veltman 1996). It **tests** whether it is consistent to accept ϕ in c. Inconsistency ($\varnothing$) results if it is not. Otherwise, c remains as it was.

The classical concept of truth is still definable in this framework, though it is a special case of the more general concept of **support**.

Definition 2 (Support, Truth in w)
(1) Support $c \vDash \phi \Leftrightarrow c[\phi] = c$ (2) Truth in w $w \vDash \phi \Leftrightarrow \{w\}[\phi] = \{w\}$

Some information c supports a sentence just in case the semantic effect of that sentence on c is *informationally redundant*. Truth in a world is a special case of support. A sentence is true in w just in case it is redundant with respect to *perfect information* about w: $\{w\}$.[19] Think of c as the content of an agent's doxastic state. Support tracks when that agent is already committed to accepting ϕ. In the extreme case where the agent has a complete picture of w, support says something unique about w. If this picture is really a complete picture of w and ϕ is already part of it, ϕ must be true in w.

The propositional content of a sentence is the set of worlds where it is true and hence determined by and distinct from its linguistic meaning (its ICP).

Definition 3 (Propositional Content) $[\![\phi]\!] = \{w \mid w \vDash \phi\}$

This method for deriving truth-conditions from ICPs will be central in §3.2.

A sentence's truth-conditions deliver a limited picture of its meaning: how it affects perfect information about the world. Its ICP delivers a broader picture: how it interacts with even uncertain information about the world. For the Boolean connectives alone there is no difference of importance here, but epistemic $\Diamond$ changes things. $\Diamond\phi$ is distinguishable from ϕ only when their effects on *imperfect information* are compared. Definitions 1.1 and 1.4 entail that $\{w\}[\Diamond \mathsf{p}] = \{w\}$ just in case $\{w\}[\mathsf{p}] = \{w\}$. Now suppose p is true in w and false

[19] This definition is mentioned by van Benthem *et al.* (1997: 594). [reference suppressed] discusses my preference for this one over Veltman's (1996: 231).

in w', but that it is uncertain which one is realized. Then $\{w, w'\}[\Diamond \mathsf{p}] = \{w, w'\}$ while $\{w, w'\}[\mathsf{p}] = \{w\}$. By contrast, the orthodox approach to distinguishing $\Diamond\phi$ and ϕ does so truth-conditionally. On the orthodox analysis $\Diamond\phi$ expresses a context-sensitive proposition reporting the fact that ϕ is consistent with some salient information in the utterance context (e.g. DeRose 1991: 593-4). But there are problems with this report-model (Yalcin 2008), ones solved by the semantics above (Yalcin 2008: §2.6).[20] On that semantics, an assertion of $\Diamond\phi$ does not convey something about the relevant information by asserting a 'second-order' proposition about that information. Rather, in interpreting an utterance of $\Diamond\phi$ the hearer's competence with $\Diamond\phi$ together with the assumption that the speaker is not being inconsistent allows the hearer to draw an inference: that the information the speaker intended to be interpreted against leaves open a ϕ-possibility. Epistemic modals are very flexible in the information they allow this to be: the participants' shared information, the speaker's private information, the hearer's private information, some expert's information and other options (von Fintel & Gillies 2007, 2008: §9). This is relevant to the semantics for indicative conditionals offered in §2.5.

In §2.5 indicative conditionals are analyzed with the same kind of 'test' semantics as $\Diamond$. They are therefore claimed to be the same kind of vehicle for indirectly conveying information as $\Diamond$. This is worth noting for two reasons. It is precisely the sensitivity to and capacity for conveying private information that has provided the strongest case for suppositional theories (e.g. Stalnaker 2005: §3). I will return to this topic in §3.3. This 'test' semantics entails that, like $\Diamond$, truth-conditions give an incomplete picture of their meaning. A complete picture must say how they interact with imperfect information too. §3.1 contends that making entailment sensitive to this dimension of meaning yields a better logic than truth-conditional and probabilistic approaches.

Definition 4 (Entailment v1) $\phi_1, \ldots, \phi_n \vDash \psi \Leftrightarrow \forall c : c[\phi_1] \cdots [\phi_n] \vDash \psi$

It says that ψ is entailed by a sequence of premises just in case adding those

[20] Yalcin (2008) goes on to give a different analysis. He notes that Veltman's semantics cannot distinguish *For all Frank believes, it's raining in Topeka* and *Frank believes it might be raining in Topeka.* Yet Veltman's analysis can be augmented with the partition structure at the heart of Yalcin's analysis to capture the contrast between these sentences. This point is elaborated below in Remark 3, Appendix A.

premises incrementally to any body of information makes ψ redundant.[21] This specifies which linguistic inference moves may be made while preserving even uncertain information. Predictably, classical entailment emerges by focusing on perfect information: $\phi_1, \ldots, \phi_n \vDash_{CL} \psi \Leftrightarrow \forall\{w\} : \{w\}[\phi_1]\cdots[\phi_n] \vDash \psi$.[22]

Ultimately, transitions between bodies of information won't cut it. To specify the meaning of declarative and interrogative sentences in one theory, the transitions will need to be between bodies of information *and issues*, i.e. $C[\phi] = C'$. The semantics in Definition 1 can be easily restated in this format (see Appendix A, Definition 14). The meaning of a polar interrogative $?\phi$ can then be specified as partitioning the worlds that would survive an update with ϕ from the worlds that wouldn't. For example, let p := 'a is a polygon', t := 'a is a triangle' and c := 'a is a circle'. Updating $C = \{\{w_0, w_1, w_2, w_3\}\}$ with $?\mathsf{p}$ will return $\{\{w_0, w_1\}, \{w_2, w_3\}\}$, pictured in Fig.3. Updating that body of information and issues with $?(\mathsf{t} \vee \mathsf{c})$ will yield the one depicted in Fig.4. My proposal to treat *if* as a polar interrogative operator amounts to saying that it does what $?$ does, namely partition the contextual possibilities.

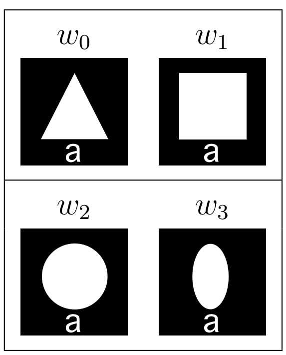

Fig. 3. $C[?\mathsf{p}]$

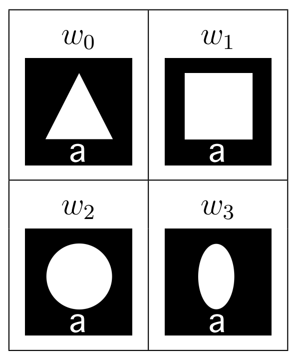

Fig. 4. $(C[?\mathsf{p}])[?(\mathsf{t} \vee \mathsf{c})]$

This section has described at length what it means to specify a semantics in terms of transitions between bodies of information and issues. But it has done little to advocate for that view, and has left out an essential ingredient of the informal analysis of conditionals I proposed in §2.1. According to that story, the interpretation of a conditional involves (i) *hypothetically* adding

[21] More on this definition: van Benthem (1996: Ch.7) and Veltman (1996: §1.2).
[22] Perfect information eliminates order-sensitivity, i.e. (i) $\{w\}[\phi_1]\cdots[\phi_n] \vDash \psi$ is equivalent to (ii) if $\{w\}[\phi_1] = \{w\}, \ldots, \{w\}[\phi_n] = \{w\}$ then $\{w\}[\psi] = \{w\}$.

the question *p?* to issues under consideration, (ii) further *hypothesizing* a p outcome and (iii) concluding that q follows from this outcome. I have said nothing about what it means to *hypothetically* adopt a question or proposition. The next section fills this gap. I will describe (some of) the transitions found in suppositional discourse and a theoretical model for understanding them. In §2.5 I will show that these transitions can be used to specify the semantic composition of conditionals, i.e. the steps informally outlined in (i)-(iii). It is only by composing meanings with this dynamic structure that the proposed semantics is able to account for the conditional-interrogative link. This, along with the discussion of entailment and truth-conditions in §3, constitutes my evidence for the kind of semantic theory outlined above.

2.4 Supposition and 'Hypothetical Additions'

Supposition exhibits a virtuosic twist on assertion and acceptance. It involves an experimental addition to the information being taken for granted. This addition is not the reflex of accepting new information, but merely entertaining it to see the landscape from a more informed perspective. The result is a kind of inquiry within an inquiry. The true virtuosity comes in how the results of this experiment in logical tourism are exported back home. To model this phenomena, I will amend the idea that the state of an inquiry or conversation is fully specified by its current background of information and issues. This amended specification should allow one inquiry to be 'nested' inside another while keeping separate the information taken for granted from the information that is merely entertained.[23] Below, I sketch just such a specification and describe how it models three transitions in suppositional discourse that will be part of the semantics for conditionals offered in §2.5.

Begin in a **state** of conversation or inquiry s where there is a lone body of contextual possibilities c, ignoring issues for simplicity. I will represent this as the unit sequence containing c: $s = \langle c \rangle$. An ordinary update with p will affect c: $s[\mathsf{p}] = \langle c[\mathsf{p}] \rangle$. Accordingly, this kind of update changes what's accepted. The **supposition** of p is a different kind of update which doesn't change c, but involves **entertaining** an update with p. This can be modeled as replicating s and updating it with p while leaving c untouched: $s' = \langle c, \langle c[\mathsf{p}] \rangle \rangle$. The left position is reserved for the contextual possibilities, while entertained

[23] See Appendix A.2. Related proposal: Kaufmann (2000).

enrichments of it are stored to the right. I call the transition of creating a hypothetical state and updating it **Subordination**, notating it $s \downarrow \mathsf{p}$. In suppositional discourse Subordination can be exploited by two other transitions. **Elaboration**, written $s' \Downarrow \mathsf{q}$, continues enriching the newly supposed information, e.g. $s' \Downarrow \mathsf{q} = \langle c, \langle c[\mathsf{p}][\mathsf{q}] \rangle \rangle = s''$.[24] **Conclusion** is the virtuosic transition that brings the results of the hypothetical inquiry to bear on what's actually being taken for granted. To see how this works, and to illustrate these other transitions in the wild, it is helpful to look at an example.

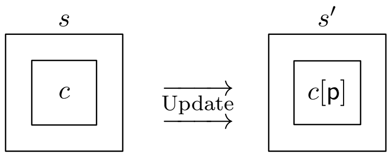

Fig. 5. Update $s[\mathsf{p}]$

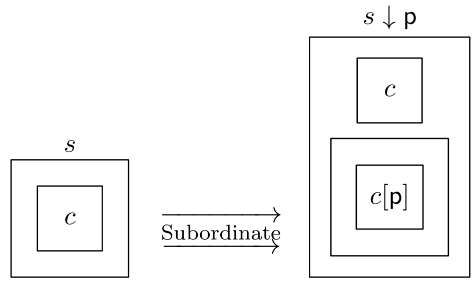

Fig. 6. Subordination $s \downarrow \mathsf{p}$

X and Y invited Paula, Quine and Roger to a potluck without telling them what to bring. Y is worries that if Paula brings a side-dish, the ratio of side-dishes to main-dishes will be wrong. X is attempting to assuage this worry.

(13) a. X: Suppose Paula brings a side-dish to the potluck.
b. Y: And let's suppose Quine brings a complementing side-dish too.
c. X: Okay. Well, then Roger will bring a main-dish, since Quine always tells Roger what to bring and Quine likes a balanced meal.
d. Y: I see. If Paula and Quine bring side-dishes, Roger will bring a main-dish.

The effect of X's accepted supposition is an instance of subordination. X and Y are entertaining the consequences of updating c with the sentence p := 'Paula brings a side-dish': $s \downarrow \mathsf{p} = \langle c, \langle c[\mathsf{p}] \rangle \rangle$. Y then goes on to elaborate this supposition with q := 'Quine brings a complementing side-dish too', yielding: $(s \downarrow \mathsf{p}) \Downarrow \mathsf{q} = \langle c, \langle c[\mathsf{p}][\mathsf{q}] \rangle \rangle$. (13c) is the crucial step. There are two observations that must be accounted for. First, whatever (13c) does together with (13a-b), licenses the indicative conditional in (13d). Second, when X commits to there

[24] Subordinating q would create yet another sub-context: $\langle c, \langle c[\mathsf{p}], \langle c[\mathsf{q}] \rangle \rangle \rangle$.

being some $\mathsf{p} \wedge \mathsf{q}$-worlds in c that are not r-worlds (13c)'s analog (14d) sounds contradictory:

(14) a. Y: Paula, Quine and Roger might all bring side-dishes!
b. X: That's true, but suppose Paula brings a side-dish.
c. Y: And let's suppose Quine brings a complimenting side-dish too.
d. X: # Okay. Well, then Roger will bring a main-dish, since Quine always tells Roger what to bring and Quine likes a balanced meal.

This example also shows that (13c)'s effects are not isolated to the suppositional context: the inconsistency of (14d) is *real* not just *entertained* as in a proof by contradiction. These two facts can be accounted for by saying that the effect of (13c) is to perform a kind of **entailment test** with r := 'Roger brings a main-dish': proceed with what you are accepting if what's supposed — $c[\mathsf{p}][\mathsf{q}]$ — entails r, otherwise fail (inconsistency). This can be captured in an equation. Where s' is the conversational state *after* (13c):

(15) $s' = \langle \{w \in c \mid c[\mathsf{p}][\mathsf{q}] \vDash \mathsf{r}\}, \langle c[\mathsf{p}][\mathsf{q}]\rangle\rangle$

It may take a second glance, but this amounts to (16).

(16) $$s' = \begin{cases} \langle c, \langle c[\mathsf{p}][\mathsf{q}]\rangle\rangle & \text{if } c[\mathsf{p}][\mathsf{q}] \vDash \mathsf{r} \\ \langle \varnothing, \langle c[\mathsf{p}][\mathsf{q}]\rangle\rangle & \text{otherwise} \end{cases}$$

I call the entailment test by which s'' arose *Conclusion*, wherein what's entertained is related to what's accepted. It is symbolized with the up arrow: $((s \downarrow \mathsf{p}) \Downarrow \mathsf{q}) \uparrow \mathsf{r} = s'$. When this test is passed in (13) it guarantees that all of the $\mathsf{p} \wedge \mathsf{q}$-worlds in c are r-worlds. This is just the condition imposed by a (epistemically) strict-conditional $\Box((\mathsf{p} \wedge \mathsf{q}) \supset \mathsf{r})$ ranging over c. On the plausible assumption that this strict-conditional will entail the corresponding indicative conditional (interpreted in c), it is clear why (13a-c) entails (13d).

2.5 The Theory

The previous section introduced three transitions between states and identified them in discourse. In this section I will propose that the same three transitions are present in the compositional semantics of conditionals.[25] Indeed, I will show that they can be used to characterize the three steps provided in the informal analysis of conditionals proposed in §2.1. That analysis claimed that a conditional is interpreted by: (i) hypothetically taking an interest in deciding between p and *not-p*, i.e. the question *p?*, (ii) further hypothesizing a p outcome and (iii) concluding that q follows from this outcome. This proposal was elucidated with a rough paraphrase of (1).

(1) If Bob danced, Leland danced
(1′) a. Suppose we are wondering if Bob danced…
  b. …and it turns out that he did.
  c. Then it will follow that Leland danced.

The interpretation of (1) starts in an initial state s and proceeds through steps resembling (1′). This process is depicted, left-to-right, in Fig. 7. First, $\mathsf{if}\ \phi$ adds

$$s[(\mathsf{if}\ \phi)\ \psi] = ((s \downarrow \mathsf{if}\ \phi) \Downarrow \phi) \uparrow \psi \text{ (preliminary version)}$$

$$c' = \{w \in c \mid c[\phi] \vDash \psi\} = c \text{ or } \varnothing$$

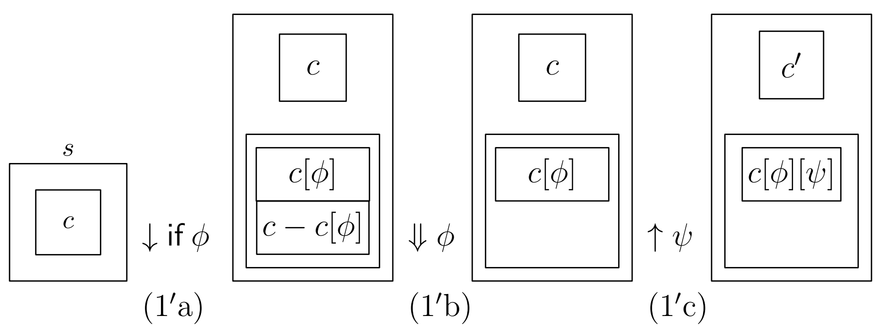

Fig. 7. Step-by-Step Effects of Conditional Update

to a *hypothetical* stock of issues, as in (1′a). Formally, this involves partitioning c into the ϕ-worlds and the $\neg\phi$-worlds, but in a hypothetical context. It is achieved by Subordinating the interrogative meaning of the *if*-clause. Second, a *yes* resolution of this issue is hypothesized, as in (1′b). This is achieved by Elaborating the affirmative answer. Third, ψ is drawn as a conclusion of this

[25] This requires taking meanings to be transitions between states. The definitions from §2.3 can be carried over to this new format: Appendix A.3.

resolution, as in (1′c). The step tests that the hypothetical information, now $c[\phi]$, makes an update with ψ informationally redundant, i.e. entails ψ. This is achieved using the Conclusion operation. Chaining these operations together:

(17) $((s \downarrow \mathsf{if}\,\phi) \Downarrow \phi) \uparrow \psi = \langle\{w \in c \mid c[\phi] \vDash \psi\}, s[\mathsf{if}\,\phi][\phi][\psi]\rangle$

If ψ is informationally redundant in $c[\phi]$ then c stays as it is, having guaranteed that all of the ϕ-possibilities in c are ψ-ones. Otherwise, a contradictory constraint has been placed on the contextual information and negotiation should ensue. With one provision to follow about the presuppositions of indicative conditionals, (17) gives the meaning of $(\mathsf{if}\,\phi)\,\psi$. Before discussing the issues surrounding that provision, I want to be more explicit about how this semantics accommodates the conditional-interrogative link.

In the conditional semantics above, *if* contributes a unary polar interrogative operator $(\mathsf{if}\,\cdot)$. There is an additional contribution made by the *syntax* of conditionals, which is the complex function built out of the arrow functions. On this view, the syntax of conditionals grammatically enforces the kind of discourse relations witnessed in advertising conditionals (§2.1) and certain suppositional discourses, e.g. (13) of §2.4. Here, I assume that the *if*-clause is an interrogative complementizer phrase adjoined to the consequent clause. So the composition rule governing its semantics is a general mechanism for combining interrogative adjuncts with a matrix clause.[26] Sentences like *Cooper wonders if Bob danced* do not have this syntactic structure, since the *if*-clause occurs as the argument of the verb *wonder*. Accordingly, the transitions involving hypothetical additions are entirely absent in them.

One might still wonder how to formulate Hamblin's semantics for embedded interrogatives in the format above. The term $\mathsf{if}\,\phi$ partitions a set of possibilities into the ϕ ones and $\neg\phi$ ones, on analogy with Hamblin's picture. Yet recall that on Hamblin's picture the semantics of a sentence like *A wonders if* ϕ involves a relation between an agent A and a question $[\![\mathit{if}\ \phi]\!] = \{[\![\phi]\!], [\![\neg\phi]\!]\}$. The compositional semantics offered above does not deal in semantic values like $[\![\mathit{if}\ \phi]\!]$, but rather in processes which divide a space of possibilities in

[26] This is rendered more plausible by noting that it offers a new direction for analyzing certain constructions that have been classified as free-relatives (Caponigro 2004), i.e. *Whether or not Bob danced, Leland danced*; *When Bob danced, Leland danced*; *Where Bob danced, Leland danced*; *How Bob danced, Leland danced.*

the corresponding way. So Hamblin's semantics for embedded interrogatives cannot be directly adopted here to unify both occurrences of *if*. However, the basic idea of that semantics can be recaptured in the present framework.

Begin by assigning each agent A in each world w a body of information and issues C^w_A representing their private agenda in inquiry, i.e. a space of epistemic possibilities partitioned into the issues A is out to settle in w. Following Hintikka (1962) and many others, attitude verbs can be represented with a relative modality for each agent, e.g. $\mathsf{B_A}(\,\cdot\,)$ for A *believes*. For *wonder* I introduce $\mathsf{W_A}(\,\cdot\,)$. Updating a state s with $\mathsf{W_A}(\mathsf{if}\,\phi)$ will eliminate any world w where either C^w_A entails ϕ or updating C^w_A with $\mathsf{if}\,\phi$ introduces some issues not already represented in C^w_A, i.e. $\langle C^w_A\rangle[\mathsf{if}\,\phi] \neq \langle C^w_A, \ldots\rangle$. Further, $\mathsf{if}\,\phi$ will presuppose that for each world w among the contextual possibilities, ϕ is compatible with C^w_A, i.e. $\langle C^w_A\rangle[\phi] \neq \langle \varnothing, \ldots\rangle$.[27] This endows $\mathsf{W_A}(\mathsf{if}\,\phi)$ with the following truth conditions: it is true in w if A's epistemic possibilities in w leave open ϕ and are already partitioned in the way accepting $?\phi$ would partition them. This is only a sketch of an analysis, but it should make clear that Hamblin's basic approach to embedded interrogatives can be maintained in the present framework. That sketch incorporated an unremarked assumption: $\mathsf{if}\,\phi$ presupposes that it is being interpreted in a context that is compatible with ϕ. This is the provision I delayed discussing three paragraphs back.

Indicative conditionals presuppose that their antecedents are possible with respect to the contextual possibilities, i.e. compatible with those possibilities. (18) exemplifies this generalization.[28]

(18) # Bob never danced. If Bob danced, Leland danced.

Many have incorporated this presupposition into their theory by stipulating that it is part of the meaning of *indicative conditionals* (Stalnaker 1975: §3; von Fintel 1999; Gillies 2009: 346). Accounting for the conditional-interrogative link leads to a more satisfying approach. This presupposition can be derived from the presupposition of the *if*-clause together with well-known facts about presupposition projection. Further, the presupposition of the *if*-clause emerges as a natural component of an interrogative semantics for *if*.

[27] For more details see Appendix A.3.

[28] To most informants, (18) was highly infelicitous. Since failed presuppositions are routinely accommodated in discourse interpretation, one would expect some noise.

It is well-known that the presuppositions of *if*-clauses project out of conditionals (e.g. Karttunen 1973: 172), so conditionals should presuppose whatever their *if*-clauses do. Hence, *q if p* should presuppose the possibility of p if the *if*-clause does. An interrogative semantics of the sort proposed here helps make sense of why *if p* should presuppose the possibility of p. First, *if* is a polar interrogative operator, so it divides the contextual possibilities into two incompatible alternatives, the p-worlds and the *not* p-worlds. If there are no p-worlds in c, this division idles. This story seems promising, but so far it fails to explain why *if*-clauses don't also presuppose that *not p* is compatible with the contextual possibilities.[29] Fortunately, there is something to say about this. *If* isn't just a polar interrogative operator, it is a biased one. More specifically, it is an interrogative operator that encodes a certain bias towards the positive answer p (Bolinger 1978; Eckardt 2007). It is a small leap to suggest that this bias also impacts the presuppositional content of *if*-clauses. But what exactly is this 'bias'. Is there any independent evidence of it?

When I say that *if p* is *biased towards the positive answer*, I mean that the positive answer is understood to be particularly relevant to some concern, decision, plan, opinion, etc.[30] This kind of bias seems promising to explain two differences between *if* and *whether*. First, *q if p* says that q is true when p is true, yet *q whether or not p* (in some dialects: *q whether p*) says that q is true both when p is true and when *not-p* is true. Thus the latter, in effect, expresses two conditionals. This is puzzling given that both *if* and *whether* are used as interrogative operators. One way of explaining it is to hold that both *if p* and *whether (or not) p* compositionally contribute two propositions, but the former highlights the positive alternative p, while the latter highlights both. A slight variation of compositional rule proposed for conditionals above can then be used in both constructions: when interpreting an interrogative clause adjoined to a matrix clause, check whether hypothetically adopting each of the 'highlighted' answer(s) to the interrogative clause entails adopting

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[29] One might argue that *if*-clauses *do* presuppose that some *not* p-worlds are live.

(19) ? Bob always danced. If Bob danced, Leland danced.

This is odd, but the intuition is less robust than (18). This might be the influence of logic training where one says things like (19) in the process of performing modus ponens. Only a thorough study of untrained intuitions could settle this.

[30] And *not* that the positive answer is itself desired or expected or preferred.

the matrix clause.[31] Second, several embedding verbs which intuitively require the alternatives to have equal status, sound marked with *if* but natural with *whether*.[32]

(20) a. Al is agonizing over whether Lily likes him
 b. ?? Al is agonizing over if Lily likes him

(21) a. Al is weighing whether he should invest
 b. ?? Al is weighing if he should invest

(22) a. Jack and Jill are disputing whether God exists
 b. ?? Jack and Jill are disputing if God exists

Now imagine you were interviewed by a journalist about your feelings on picking up hitchhikers. Which of the below best reports this?

(23) a. The reporter came up to me and asked if I would give him a lift
 b. The reporter came up to me and asked whether I would give him a lift

It seems that (23b) does. (Bolinger 1978: 96) (23a) sounds more suitable as a report of actually being propositioned by a hitchhiker. Finally, Eckardt (2007) observes that (24) supports a similar generalization.

(24) a. We need to find out who the first speaker is and if she needs a projector.
 b. We need to find out who the first speaker is and if she doesn't need a projector.
 c. We need to find out who the first speaker is and whether she needs a projector.

Among conference planners, (24a) seems appropriate where special arrangements would be required if the first speaker needs a projector but not otherwise. (24b) seems appropriate where it would be helpful if the first speaker *didn't* need a projector, say because one is needed in another room. (24c) does not seem to portray either answer as more relevant to the organizers'

[31] This would also provide an analysis of what Caponigro (2004) calls *prepositional phrase free relatives*, like *Bob dances where Leland dances*, *Bob dances how Leland dances* and *Bob dances when Leland dances*.

[32] Bolinger (1978: 93) discusses similar data.

planning process. Together, this evidence suggests that treating *if* as a biased interrogative operator is independently motivated. I therefore propose that the interrogative theory of *if* helpfully explains what other theories assume: *if p* presupposes the epistemic possibility of p.

This paper will not attempt a formal analysis that does justice to the above details of presupposition projection and interrogative bias. However, it will be important to include their upshot in the semantics, namely that indicative conditionals presuppose that their antecedent is compatible with c. The following adaptation of (17) suffices for this purpose.

Definition 5 (Inquisitive Conditional Semantics)

$$s[(\mathsf{if}\,\phi)\,\psi] = \begin{cases} ((s \downarrow \mathsf{if}\,\phi) \Downarrow \phi) \uparrow \psi & \text{if } s[\phi] \neq \langle \varnothing, \ldots \rangle \\ \text{Undefined} & \text{otherwise} \end{cases}$$

This definition treats presupposition failure as undefined update. Other authors have developed this idea at length (e.g. Heim 1983; Beaver 2001). In §3 it will become clear why it is important to represent this nuance in the semantics offered here.

3 A New Look at Old Issues

The semantics in §2.5 does more than accommodate the conditional-interrogative link. It provides a new take on familiar issues in the study of conditionals.

3.1 The Logic of Indicative Conditionals

Consider the two worst entailments of the material conditional:

(25) **Material Antecedent (MA)** $\neg\phi \vDash \phi \supset \psi$
Bob didn't dance. So, if Bob danced, he was a turnip.

(26) **Material Negation (MN)** $\neg(\phi \supset \psi) \vDash \phi$
It's not true that if God exists, he's a turnip. So, God exists.

Stalnaker (1968, 1975) and Adams (1975) propose indicative conditionals that invalidate MA and MN, but do so at the cost of invalidating:[33]

(27) **Import-Export** $\phi_1 \to (\phi_2 \to \psi) \dashv\vDash (\phi_1 \wedge \phi_2) \to \psi$
Antecedent Strengthening $\phi_1 \to \psi \vDash (\phi_1 \wedge \phi_2) \to \psi$
Disjunctive Antecedents $(\phi_1 \vee \phi_2) \to \psi \vDash (\phi_1 \to \psi) \wedge (\phi_2 \to \psi)$
Transitivity $\phi_1 \to \phi_2, \phi_2 \to \psi \vDash \phi_1 \to \psi$
Contraposition $\phi \to \psi \vDash \neg\psi \to \neg\phi$

Instances of these principles in natural language generally sound good, but Adams and Stalnaker have artfully constructed a few that do not. The semantics offered in §2.5 can validate all of the patterns in (27), expose the craft of these alleged counterexamples and handle (25) and (26).[34] I will defend this claim in detail below. The pattern of explanation and logic that emerges is reminiscent of sophisticated strict-conditional theories (Warmbrod 1983: §5; Gillies 2009: 338, 347). The explanation has two key features. The first is that entailment is thought of in terms of general information preservation rather than truth preservation, as in §2.3. The second is that the presuppositional behavior of indicatives (§2.5) is taken into account — much as Strawson (1952: 173-9) suggested for *all*. Both features were motivated by the account of the conditional-interrogative link above. So what emerges appears to be a more general and streamlined account of *if* and the logic of indicative conditionals that it engenders.

[33] McGee (1989) extends Adams' approach to handle some embeddings and validate import-export. But it still does not yield a plausible semantics of natural language conditionals (Edgington 2008: §4.3). Stalnaker (1975) introduces the notion of a *reasonable inference* which offers some consolation. Contraposition and transitivity come out as reasonable inferences — the former requiring the additional assumption that the conclusion is appropriate in the context augmented with the premises (Stalnaker 1975: §IV). But this does not provide any help with import-export, antecedent strengthening or disjunctive antecedents.

[34] One caveat is in order. Only restricted versions of the final four patterns in (27) are valid when conditionals involving embedded conditionals and modal expressions are allowed. However, I detail in Appendix A.5.2 intuitive counterexamples to the unrestricted versions. In many cases, these are novel counterexamples not discussed elsewhere in the literature.

Entailment is about information. So to study the logic of inquisitive conditionals, it is only necessary to attend to the way they affect the contextual possibilities. The following fact describes this effect completely.

Fact 1 (The Inquisitive Conditional is Strict over c)
Let s be a state and c_s the contextual possibilities in that state. Then the effect of $(\mathsf{if}\,\phi)\,\psi$ on c_s is identical to the following update just defined on c_s:

$$c_s[(\mathsf{if}\,\phi)\,\psi] = \begin{cases} \{w \in c_s \mid c_s[\phi] \vDash \psi\} & \text{if } c_s[\phi] \neq \varnothing \\ \text{Undefined} & \text{otherwise} \end{cases}$$

Proof See Appendix A.5.2.

If ϕ is compatible with c_s, an update with $(\mathsf{if}\,\phi)\,\psi$ will test whether all of the ϕ-worlds in c_s are ψ-worlds. This test returns the whole set if passed and returns none of it if failed. If ϕ is incompatible with c_s, the update is simply undefined. This semantics has the flavor of a strict-conditional semantics, and something quite like it is advanced by Warmbrod (1983: §5) and Gillies (2009: 338, 347). As they note, it can provide a very successful logic for indicative conditionals. Modus ponens and modus tollens are valid and $(\mathsf{if}\,\phi)\,\phi$ is a logical necessity, but what about the forms in (27)?

Neither MA nor MN are valid for the inquisitive conditional (on Definition 4). Consider MA. Given Definition 4 for entailment, MA claims that following an update with $\neg\phi$ and update with $(\mathsf{if}\,\phi)\,\psi$ is redundant, i.e. $c[\neg\phi] = c[\neg\phi][(\mathsf{if}\,\phi)\,\psi]$. $c[\neg\phi]$ eliminates all ϕ-worlds. But, any update with $(\mathsf{if}\,\phi)\,\psi$ presupposes that there is at least one ϕ-world, so $c[\neg\phi][(\mathsf{if}\,\phi)\,\psi]$ is *never defined* let alone always identical to $c[\neg\phi]$. MN requires $c[\neg((\mathsf{if}\,\phi)\,\psi)] = c[\neg((\mathsf{if}\,\phi)\,\psi)][\phi]$. Consider the left term. By the semantics of negation $c[\neg((\mathsf{if}\,\phi)\,\psi)] = c - c[(\mathsf{if}\,\phi)\,\psi]$. Let c be any set flunking the test invoked by $(\mathsf{if}\,\phi)\,\psi$, but containing some $\neg\phi$ world w. Then $c[\neg((\mathsf{if}\,\phi)\,\psi)] = c - \varnothing = c$. Thus MN's requirement comes to $c = c[\phi]$. This cannot be true: $w \in c$ and $w \notin c[\phi]$. Hence $\neg((\mathsf{if}\,\phi)\,\psi) \nvDash \phi$. The validity of the patterns in (27) is demonstrated in Appendix A.5.

Consider Bennett's (2003: 145) allegation that transitivity is invalid. A farmer is reflecting on the state of his farm. He is nearly certain that the cows are not in the field, but accepts that

(28) a. If the cows are in the field, the gate is open

Since he's nearly certain that they aren't in the field he also accepts:

(28) b. If the gate is open, the cows haven't noticed it

By transitivity, it would seem to follow that

(28) c. If the cows are in the field, they haven't noticed that the gate is open

but this sounds wrong. What gives? On the theory advanced here (28a) tests that all the live in-the-field worlds are gate-is-open worlds and presupposes that this test is not vacuously passed, i.e. there is at least one live in-the-field world w. So, in w the cows are in the field and the gate is open, presumably they got there by noticing this. Now turn to (28b). This requires that all the gate-open worlds are haven't-noticed worlds. But this requirement will not be passed on account of w. The impression that the farmer can consistently accept (28a) and (28b) can only come from forgetting (28a)'s presupposition and letting (28a) be vacuously satisfied. The inference sounds strange, just like any unexpected inferential leap from inconsistent premises. Hence (28) only shows that transitivity may lead us astray when we forget the presuppositions of accepting a conditional. This is a reminder, not a counterexample.

Now consider contraposition. From *if it rains, there won't be a terrific cloudburst* it doesn't seem to follow that *if there is a terrific cloudburst, it won't rain* (Adams 1975: 15).[35] The premise tests that all of the live rain worlds are not-cloudburst worlds. The contrapositive *if there is a terrific cloudburst, it won't rain* presupposes that there is a live cloudburst world. Cloudbursts require rain, so this must be a rain world. But the premise required that all of the live rain worlds be *not*-cloudburst worlds. Hence, the conclusion's presupposition cannot be met in any c where the premise can be accepted. As the theory has been stated, this *is* a counterexample to contraposition. The equality $c[(\mathsf{if\ r})\,\neg\mathsf{c}] = c[(\mathsf{if\ r})\,\neg\mathsf{c}][(\mathsf{if\ c})\,\neg\mathsf{r}]$ fails since the left term may be defined while the right is not. But one may respond by following Strawson (1952: 173-9) who maintains that only cases where presuppositions are met count against validity. This amounts to modifying the definition of entailment: if the presuppositions of the premises and conclusion are incrementally satisfied then the

[35] Gillies (2009: 346-8) also discusses these examples involving contraposition.

premises make the conclusion informationally redundant.

Definition 6 (Entailment v2)
$\phi_1, \ldots, \phi_n \vDash \psi \Leftrightarrow \forall c :$ if $c[\phi_1] \cdots [\phi_n][\psi]$ is defined, $c[\phi_1] \cdots [\phi_n] \vDash \psi$

It is neither implausible nor ad-hoc to say that language users do not count cases involving presupposition failure against an otherwise valid argument. Indeed, on further inspection this modification is already required to secure other validities. Consider an instance of modus ponens: $\mathsf{p}, (\mathsf{if\ p})\,((\mathsf{if\ q})\,\mathsf{r}) \vDash (\mathsf{if\ q})\,\mathsf{r}$ and a c containing a $\mathsf{p} \wedge \mathsf{q} \wedge \neg\mathsf{r}$-world. $c[\mathsf{p}][(\mathsf{if\ p})\,((\mathsf{if\ q})\,\mathsf{r})] = \varnothing$ yet $\varnothing[(\mathsf{if\ q})\,\mathsf{r}]$ is undefined. So it's not true that for every $c : c[\mathsf{p}][(\mathsf{if\ p})\,((\mathsf{if\ q})\,\mathsf{r})] = c[\mathsf{p}][(\mathsf{if\ p})\,((\mathsf{if\ q})\,\mathsf{r})][(\mathsf{if\ q})\,\mathsf{r}]$. It's only true for contexts where the presuppositions are satisfied. Similarly, $\vDash (\mathsf{if}\ \phi)\,\phi$ would fail since you could always find a c incompatible with ϕ. Alleged counterexamples to antecedent strengthening and disjunctive antecedents employ the same trick Adams did above: a move from a satisfied premise to an unsatisfied conclusion. Hence, by adopting the more sophisticated definition of entailment above one is able to provide a tidy explanation of why the forms in (27) generally seem valid and say exactly what is going on in the cases that have been offered as counterexamples.

The move to Definition 6 requires reconsidering the diagnosis of MA above. Since MA is a case where the presuppositions are *never* met, it vacuously satisfies Definition 6. One could amend Definition 6 to also require that there is a c where the presuppositions of the premises and conclusion can be incrementally satisfied. This would have the consequence of invalidating any instances of any arguments containing conditionals with logically inconsistent antecedents, e.g. $(\mathsf{if\ p} \wedge \neg\mathsf{p})\,\psi, \mathsf{p} \wedge \neg\mathsf{p} \vDash \psi$ and $\vDash (\mathsf{if\ p} \wedge \neg\mathsf{p})\,(\mathsf{p} \wedge \neg\mathsf{p})$. Since speakers have no positive intuitions about the validity of these forms, this is not a serious empirical problem. But, it yields a byzantine formal system where validities are limited to conditionals with consistent antecedents and certain desirable metatheorems fail, e.g. $\vDash (\mathsf{if}\ \phi)\,\psi \Leftrightarrow \phi \vDash \psi$. There is a more tidy alternative. Even if one grants the technical validity of MA, the semantics above does not predict than any instance of MA should sound acceptable in any context to competent speakers. After all, acceptability requires presuppositions to be met and MA cannot meet them in any context. Accordingly, one can grant vacuous validities to simplify the logic without predicting that they govern the intuitions of any speaker in any context. I propose viewing MA in this light.

I have seen no attempts to invalidate import-export, and its validity is quite uncomfortable to deny. Given this and the above, the semantics offered here deserves to be regarded as delivering the most successful logic of natural language indicative conditionals available.[36] A key part of this account was the assumption that indicative conditionals presuppose the possibility of their antecedent. As discussed in §2.5, inquisitive conditional semantics provides a more principled story about this assumption than previous accounts, hence strict-conditional ones in particular. Truth-conditions provide another point of difference between the two theories. The framework from §2.3 determines the truth-conditions of sentences from their dynamic effects on the contextual information.This method yields context-insensitive, partial truth-conditions for indicative conditionals. §3.2 will illustrate one beneficial consequence of this difference.

3.2 Truth-Conditions, Bets and Indicative Conditionals

Consider the following case. We think that the local zoo might get a new animal this spring, but have different hunches about what it would be. I suspect an armadillo and you a roadrunner. We like to bet so I wager $5 that

(29) If the zoo gets an animal this spring it will be an armadillo

You wager against me. Spring comes. It brings wildflowers, birds, bees, but no new animal to the zoo. Who has to pay? The intuition that neither of us gets paid is overwhelming. This remains even if we find out that the zoo board had decided to get an armadillo but the funding was cut at the last minute. I made the better bet, but my attempts to collect $5 may be rebuffed. You can point out that our bet only covered the case where the zoo got an animal, so the bet is off. You can even acknowledge that if the funding hadn't been cut, the zoo would have gotten an armadillo, but hold fast that technically you got lucky and are off the hook. This phenomena is problematic for traditional similarity and strict-conditional theories.

These traditional theories are wedded to the following story. When (29) is uttered in the betting-context it expresses a proposition. After all, we agree

[36] There are several points of contact between the semantics defended here and Lycan (2001). The present proposal parts way with Lycan in upholding modus ponens, transitivity and antecedent strengthening.

that the local zoo might get a new animal, so (29)'s presupposition that the antecedent is compatible with the available information is met in that context. This information will also determine exactly which proposition is expressed. When we evaluate the bet, we are evaluating the truth of this proposition in a world where the zoo didn't get an animal but if it had it would have been an armadillo. The problem is that these theories predict this proposition to be true, but our intuitions require it to be neither true nor false. On the similarity theory, we bet on the proposition that the zoo-gets-animal worlds most similar to those compatible with our information are zoo-gets-armadillo worlds. On the strict theory, we bet on the proposition that all the worlds accessible from those compatible with our information are either zoo-doesn't-get-animal worlds or zoo-gets-armadillo worlds. The actual world $w_@$ was among the worlds compatible with our information. Given the board's choice, the zoo-gets-animal worlds most similar to $w_@$ are zoo-gets-armadillo worlds. Similarly, all the worlds accessible from $w_@$ should be either zoo-doesn't-get-animal worlds or zoo-gets-armadillo worlds. So both propositions are true. But if the proposition is true then why do you get off the hook? Isn't settling our bet settling the truth-value of the proposition on which we bet?

A traditional theorist might attempt to respond by pointing out that (29)'s presupposition is no longer met in the context where the bet is being settled. This response would liken our bet on (29) to a case where we bet on

(30) Carlotta stopped smoking

only to find out that she never smoked to begin with. Here too it seems that the bet would be called off. There is still a puzzle if one's semantics predicts (30) to be true in such a scenario, but bracketing this issue there is an important difference between the cases. In betting on (30) we presupposed that Carlotta smoked at some point and then find out that this presupposition was *false*. This is not the case in (29). We presupposed that the zoo getting a new animal was compatible with our information in the betting context. We do not then find out that this presupposition was *false*, i.e. the zoo getting a new animal was *not* compatible with our information in the betting context. We just get better informed and according to the traditional semantics learn that the proposition expressed by (29) was false. Our intuitions about (29) remain unexplained.

There is a long history of developing three-valued logics to account for intuitions of this sort (e.g. de Finetti 1936: 35; Jeffrey 1963; Belnap 1973; McDermott 1996: 6; Milne 1997). Immediately below, I argue that none of these attempts are satisfactory. But first, I will focus on the positive account delivered by the ideas developed in §§2.3 and 2.5. This account will mimic the three-valued one, but with more plausible results. [37] Recall §2.3's method for deriving truth-conditions from information change potentials: ϕ is true in w iff $\{w\}[\phi] = \{w\}$. Adapting this to states, ϕ is true in w iff $\langle\{w\}\rangle[\phi] = \langle\{w\}, \ldots\rangle$. This definition yields the following consequences for the inquisitive conditional.

Fact 2 (Truth-Conditions for Inquisitive Indicative Conditionals)

(1) $(\mathsf{if}\,\phi)\,\psi$ is true in w, if both ϕ and ψ are true in w.
(2) $(\mathsf{if}\,\phi)\,\psi$ is false in w, if ϕ is true in w and ψ is false in w
(3) Otherwise, $(\mathsf{if}\,\phi)\,\psi$'s truth-value is undetermined in w
(4) Therefore, $[\![(\mathsf{if}\,\phi)\,\psi]\!] = \{w \mid w \vDash \phi\}$ is not a well-defined proposition □

Suppose ϕ is false in w. What is $(\mathsf{if}\,\phi)\,\psi$'s truth-value in w? This amounts to asking what the result of $\langle\{w\}\rangle[(\mathsf{if}\,\phi)\,\psi]$ is. This conditional update must meet the presupposition that $\langle\{w\}\rangle[\phi] \neq \langle\varnothing, \ldots\rangle$. But since ϕ is false in w, this presupposition is not met, i.e. $\langle\{w\}\rangle[\phi] = \langle\varnothing, \ldots\rangle$. Thus, $(\mathsf{if}\,\phi)\,\psi$'s truth-value is undefined at any world where ϕ is false. Similar reasoning confirms the other truth-conditions stated above in Fact 2. In the bet scenario, we ended up in a world where the conditional is undefined, hence neither true nor false. Note that if the presupposition is relaxed, $[\![(\mathsf{if}\,\phi)\,\psi]\!] = [\![\neg\phi \vee \psi]\!]$. But, this proposition would play no role in the logic or communicative use of conditionals.

Similar accounts using three-valued logics face daunting difficulties. These approaches assign $\phi \rightarrow \psi$ to the truth-value assigned to ψ, if ϕ is assigned 1 (true). Otherwise, the conditional is assigned i (indeterminate). To block Material Negation $\neg(\phi \rightarrow \psi) \vDash \phi \wedge \neg\psi$ and the even more garish $\phi \rightarrow \psi \vDash \phi \wedge \psi$, they must require that a valid argument does not merely preserve truth. It must also guarantee that if the conclusion is false at least one of the premises is false (McDermott 1996: 31). It is far from clear how such a definition should be motivated, but setting this aside, more concrete difficulties emerge. Modus

[37] There is one important point of contrast not elaborated below. Since the partiality of my theory is aligned with presupposition failure, it may appeal to presupposition accommodation to account for cases where one is tempted to say that an indicative conditional has a truth-value at a world where its antecedent is false.

ponens becomes invalid, since $(\phi \to \phi) \to \phi, \phi \to \phi \vDash \phi$ comes out invalid. When the conclusion is false none of the premises have truth-values. These theories require profligate meanings for the other sentential connectives (e.g. McDermott 1996: 5). While suppressing Material Negation, they fail to validate an entailment that captures the intuitive point of a negated indicative conditional: $\neg(\phi \to \psi) \vDash \Diamond(\phi \wedge \neg\psi)$; when it is known that $\neg\phi$, the conclusion is false but the premise is undetermined. Three-valued accounts also invalidate contraposition. When $\neg\phi \to \neg\psi$ is false $\phi \to \psi$ will be undefined.[38] As discussed in §3.1, the semantics from §2.5 delivers a far superior logic of indicative conditionals. A further difficulty for trivalent accounts arises from the fact that they offer no clear account of the relationship between indicative and subjunctive conditionals. [reference suppressed] shows that the present theory does not suffer from this difficulty.

3.3 Open Ends

Sensitivity to Private Information

Gibbard (1981: §7) presents a case with the following form (see also Bennett 2003: §34). Gibbard receives two unsigned notes from his trusted confederates, Zach and Jack, about a poker match. One note contains the conditional *if p*

[38] Huitink (2008: §5.3) attempts to address this problem. She begins with 'Strawson Validity': $\phi_1 \vDash_S \psi$ iff $\phi_1, \phi_2 \vDash \psi$, where $\vDash$ is classical validity and ϕ_2's truth guarantees that ϕ_1 and ψ have classical truth-values. Since $\phi \to \psi, \neg\psi \vDash \neg\psi \to \neg\phi$, contraposition is Strawson Valid even on a trivalent semantics. Unfortunately, Material Negation is Strawson Valid too, since $\neg(\phi \to \psi), \phi \vDash \phi$. Even more gruesome is the result that $\phi \to \psi \vDash_S \phi \wedge \psi$, since $\phi \to \psi, \phi \vDash \phi \wedge \psi$. Even if a technical bandage can be found for these wounds, the proposal suffers from a more basic flaw. It is built on the idea that some inferences like contraposition have an implicit premise which is that all of the sentences involved have truth-values (Huitink 2008: 174). This is fine, but there is no plausibility to the claim that intuitions about the validity of contraposition rely on the implicit premise that $\neg\psi$. Consider the following line of reasoning. Bob might have danced and if he did, Leland danced. So Leland might have danced, but if he didn't, Bob didn't either. Here, contraposition sounds correct despite the fact that the antecedent of the conclusion is explicitly *not* accepted. The alternative detour through reasonable inference creates the same problems (Huitink 2008: §5.3.2). Jeffrey (1963: 39) validates contraposition by a different route but also at the cost of validating Material Negation. Much more work is needed to make the case that a trivalent account can maintain contraposition in a plausible way.

then q and is intuitively true as the case is described. The other note contains a conditional *if p then not-q* that also seems true. Propositional theories predict these two conditionals to express incompatible propositions at any context meeting their presupposition. So these theories must say what is different about the two note-issuing contexts. As Gibbard sets up the case, the common knowledge in the two contexts is the same. So any contextualist reply will appeal to a difference in context known only to one (or none) of the conversationalists. In Gibbard's case, it appears to be the private knowledge had by Zach and Jack, respectively. But this saddles both of them with defective communicative intentions. They each intend to communicate a proposition with a speech act that requires their hearer to have some collateral information to get that proposition out of that speech act. But they intend to do so while knowing that their hearer does not have that information.

The semantics from §2.5 offers a different picture. When a speaker issues *if p then q*, they intend to be interpreted against some information/possibilities c. The semantics says that this conditional will reduce c to $\varnothing$ if there is a *p-and-not-q* world in c, and leave c as it is otherwise. Knowing this, and assuming the speaker is being consistent, the hearer may infer something about c: it contains no *p-and-not-q* worlds. When c is private information, this process will get across something about the speaker's private information without assuming the hearer has access to it and without directly reporting it. Further, it works just as well for getting across information about other bodies of information, e.g. what is known by experts, what is common knowledge, etc. So the picture is not ruined by familiar cases where the speaker will rescind their assertion in light of better information (von Fintel & Gillies 2008). It is not my goal here to promote this picture over propositional approaches to Gibbard's case (e.g. Stalnaker 2005: §2). I merely observe that it looks like (an improvement on) the non-propositional alternative.

Adverbs of Quantification

Lewis (1975) argued that no plausible connective-based analysis of conditionals could yield the correct truth-conditions when embedded under adverbs with varying quantificational force, e.g. *always*, *sometimes* and *usually*. Lewis (1975: n14) acknowledged that a semantics, like Belnap's (1973), that delivers trivalent truth-conditions could produce the correct truth-conditions in these cases, but contends that such an account makes too many compromises. Lewis concludes that *if... then...* does not have its usual meaning in these constructions and instead merely serves to mark the restrictor and nuclear scope of the quantificational adverbs. Kratzer (1986, 1991) extended this argument to modal adverbs and developed the more plausible (and radical) view that all conditionals work in this way, analyzing bare conditionals as containing covert universal modal quantifiers.

Recently, Huitink (2008: Ch.5) has raised some problems for Kratzer's (1986) analysis, shown that a sophisticated version of Belnap's semantics solves these difficulties and used that semantics to account for the interaction of conditionals with quantificational adverbs. As I argued in §3.2, trivalent approaches like the Huitink/Belnap analysis make unacceptable logical compromises. However, the trivalent approach developed here avoids these drawbacks. Yet, in virtue of its trivalent truth-conditions, it can also cover the data involving quantificational adverbs. A more thorough demonstration of this is in order, but a partial demonstration follows from the semantics given in Appendix §A. On that semantics $\Box(\mathsf{if}\ \phi)\,\psi$, $(\mathsf{if}\ \phi)\,\Box\psi$ and $(\mathsf{if}\ \phi)\,\psi$ are equivalent, as are $(\mathsf{if}\ \phi)\,\Diamond\psi$ and $\Diamond(\phi \wedge \psi)$.

Conditionals and Probability

Under certain assumptions, the (conditional) probability of q given p cannot be identified with with the probability of any proposition, let alone the one that if p then q.[39] Yet, many have been attracted to these assumptions and **the Equation** $P(\textit{if } p \textit{ then } q) = P(q \mid p)$ (Adams 1975; McGee 1989; Bennett 2003; Edgington 2008) and hence adopting a non-propositional semantics. The style of semantics proposed in §2.5 displaced propositions in favor of transitions between states of inquiry. Probability calculi with this flavor have been studied (van Benthem *et al.* 2009). This offers one open prospect for maintaining the Equation. There is, however, a simple and familiar option provided by the semantics above. If the probability of a conditional is just a measure of the worlds where it is true, the three-valued truth-conditions generated by my semantics entails the equation. That such a three-valued semantics has this property has been known since de Finetti (1936). The present account offers a new and more attractive way of endorsing such a semantics.

[39] Lewis (1976, 1986); Hájek & Hall (1994).

And More
The consequences explored here in §3 have made no use of the resources provided by states of inquiry (§2.4) other than their ability to encode a body of contextual information c. However, this additional structure is used in my related work subjunctive conditionals [reference suppressed] to understand Sobel sequences and subjunctive conditional questions, e.g. *if Bob had danced would Leland have danced?*.

4 Conclusion

The conditional-interrogative link shed doubt on truth-conditional connective and suppositional approaches. Making sense of this data instigated a shift in the format of semantic theory. Instead of propositions, transitions between bodies of information and issues took center stage. The accompanying perspective on entailment and truth-conditions turned out to offer a more successful account of indicative conditionals than the best versions of either approach (§§3.2, 3.1). The basic idea of the semantics offered here has much in common with the idea behind suppositional approaches. It is unsurprising then that it holds promise for enjoying the two most desired benefits of that approach: sensitivity to private information and their ability to preserve the Equation (§3.3). Yet, this semantics also has the benefits of a truth-conditional connective approach. It offers clear predictions about truth-value judgements, adheres to compositionality, treats indicative and subjunctive conditionals uniformly and integrates with truth-conditional frameworks used for other kinds of sentences. The semantic framework presented here is a generalization of truth-conditional semantics (§2.3), so the resources and insights of that approach are preserved in this new framework.[40] Hence, the benefits of suppositional and truth-conditional approaches may not be exclusive after all. This particular case-study in semantics points to a broader research program. Having earned a place in concrete applications, the hybrid perspective on meaning it employs merits further investigation and philosophical elaboration.

[40] Muskens (1996) shows how to combine Montague's compositional framework with the kind of semantics developed here.

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A The Logic of Inquisitive Conditionals (LIC)

A.1 Syntax

Remark 1 For simplicity, assume $\mathsf{if}\,\phi := ?\phi$. Although strictly speaking a conditional is written $((?\phi)(\psi))$, I will prefer the more readable $(\mathsf{if}\,\phi)\,\psi$.

Definition 7 (LIC Syntax)

(1)	$p \in \mathcal{W}\!f\!f_A$	if $p \in \mathcal{A}t = \{\mathsf{a}_0, \mathsf{a}_1, \ldots\}$
(2)	$\neg\phi \in \mathcal{W}\!f\!f_A$	if $\phi \in \mathcal{W}\!f\!f_A$
(3)	$\Diamond\phi \in \mathcal{W}\!f\!f_A$	if $\phi \in \mathcal{W}\!f\!f_A$
(4)	$\Box\phi \in \mathcal{W}\!f\!f_A$	if $\phi \in \mathcal{W}\!f\!f_A$
(5)	$(\phi \wedge \psi) \in \mathcal{W}\!f\!f_A$	if $\phi, \psi \in \mathcal{W}\!f\!f_A$
(6)	$(\phi \vee \psi) \in \mathcal{W}\!f\!f_A$	if $\phi, \psi \in \mathcal{W}\!f\!f_A$
(7)	$(?\phi) \in \mathcal{W}\!f\!f_Q$	if $\phi \in \mathcal{W}\!f\!f_A$
(8)	$(\phi \wedge \psi) \in \mathcal{W}\!f\!f_Q$	if $\phi, \psi \in \mathcal{W}\!f\!f_Q$
(9)	$(\phi \vee \psi) \in \mathcal{W}\!f\!f_Q$	if $\phi, \psi \in \mathcal{W}\!f\!f_Q$
(10)	$\phi \in \mathcal{W}\!f\!f_{AQ}$	if $\phi \in \mathcal{W}\!f\!f_A \cup \mathcal{W}\!f\!f_Q$
(11)	$\phi \in \mathcal{W}\!f\!f_{Q+}$	if $\phi \in \mathcal{W}\!f\!f_{AQ}$
(12)	$((?\phi)(\psi)) \in \mathcal{W}\!f\!f_{Q+}$	if $\phi \in \mathcal{W}\!f\!f_A, \psi \in \mathcal{W}\!f\!f_Q$

(13)	$((?\phi)(\psi)) \in \mathcal{W}\!f\!f_C$	if $\phi \in \mathcal{W}\!f\!f_A, \psi \in \mathcal{W}\!f\!f_A$
(14)	$((?\phi)(\psi)) \in \mathcal{W}\!f\!f_C$	if $\phi \in \mathcal{W}\!f\!f_C, \psi \in \mathcal{W}\!f\!f_C$
(15)	$\neg\phi \in \mathcal{W}\!f\!f_C$	if $\phi \in \mathcal{W}\!f\!f_C$
(16)	$\Diamond\phi \in \mathcal{W}\!f\!f_C$	if $\phi \in \mathcal{W}\!f\!f_C$
(17)	$\Box\phi \in \mathcal{W}\!f\!f_C$	if $\phi \in \mathcal{W}\!f\!f_C$
(18)	$(\phi \wedge \psi) \in \mathcal{W}\!f\!f_C$	if $\phi, \psi \in \mathcal{W}\!f\!f_C$
(19)	$(\phi \vee \psi) \in \mathcal{W}\!f\!f_C$	if $\phi, \psi \in \mathcal{W}\!f\!f_C$
(20)	$\phi \in \mathcal{W}\!f\!f$	if $\phi \in \mathcal{W}\!f\!f_{Q+} \cup \mathcal{W}\!f\!f_C$

A.2 States and Operations on Them

Definition 8 (Worlds) $W : \mathcal{A}t \mapsto \{1, 0\}$ where $\mathcal{A}t = \{\mathsf{p}_0, \mathsf{p}_1, \ldots\}$

Definition 9 (Contextual Possibilities/Information) $c \subseteq W$

Definition 10 (Contextual Information and Issues)

- C is a non-empty set of subsets of W
  - $\varnothing \neq C \subseteq \mathscr{P}(W)$
- $\mathcal{C}$ is the set of all such C
- $\bigcup C$ is the information embodied by C; the sets in C are called *alternatives*; overlapping and non-maximal alternatives are allowed.

Definition 11 (States) S is the set of all states

(1) If $C \in \mathcal{C}$, $\langle C \rangle \in S$.
(2) If $C \in \mathcal{C}, s \in S$, $\langle C, s \rangle \in S$.
(3) Nothing else is a member of S.

Definition 12 (Subordination, Elaboration, Conclusion)
Where $s = \langle C, \langle C_0, \ldots \langle C_n \rangle \cdots \rangle \rangle$:
(1) $s \downarrow \phi = \langle C, \langle C_0, \ldots \langle C_n, \langle C \rangle [\phi] \rangle \cdots \rangle \rangle$
(2) $s \Downarrow \phi = \langle C, \langle C_0, \ldots \langle C_n \rangle [\phi] \cdots \rangle \rangle$
(3) $s \uparrow \psi = \langle \{c \in C \mid \langle C_n \rangle \vDash \psi\}, \langle C_0, \ldots \langle C_n, \langle C_n \rangle [\psi] \rangle \ldots \rangle \, \rangle$

A.3 Systems of Update Semantics

Definition 13 (Informational Semantics) $[\,\cdot\,] : (\mathcal{W}\!f\!f_A \times C) \mapsto C$

(1)	$c[\mathsf{p}]$	$=$	$\{w \in c \mid w(\mathsf{p}) = 1\}$	(2)	$c[\neg\phi]$	$=$	$c - c[\phi]$
(3)	$c[\phi \wedge \psi]$	$=$	$(c[\phi])[\psi]$	(4)	$c[\phi \vee \psi]$	$=$	$c[\phi] \cup c[\psi]$
(5)	$c[\Diamond\phi]$	$=$	$\{w \in c \mid c[\phi] \neq \varnothing\}$	(6)	$c[\Box\phi]$	$=$	$\{w \in c \mid c[\phi] = c\}$

Definition 14 (Inquisitive Semantics) $[\,\cdot\,] : (\mathcal{W}\!f\!f_{AQ} \times \mathcal{C}) \mapsto \mathcal{C}$

Where $C = \{c_0, \ldots, c_n\}$ and $\overline{C}_\phi := \{\, c_0 - \bigcup(\{c_0\}[\phi]), \ldots, c_n - \bigcup(\{c_n\}[\phi]) \,\}$:

(1)	$C[\mathsf{p}]$	$=$	$\{\, \{w \in c_0 \mid w(\mathsf{p}) = 1\}, \ldots, \{w \in c_n \mid w(\mathsf{p}) = 1\} \,\}$
(2)	$C[\neg\phi]$	$=$	$\overline{C}_\phi$
(3)	$C[\phi \wedge \psi]$	$=$	$(C[\phi])[\psi]$
(4)	$C[\phi \vee \psi]$	$=$	$C[\phi] \cup C[\psi]$
(5)	$C[?\phi]$	$=$	$C[\phi] \cup \overline{C}_\phi$
(6)	$C[\Diamond\phi]$	$=$	$\{c' \in C \mid C[\phi] \neq \{\varnothing\}\}$
(7)	$C[\Box\phi]$	$=$	$\{c' \in C \mid \bigcup(C[\phi]) = \bigcup C\}$

Remark 2 Above, $\overline{C}_\phi$ may be pronounced *the* ϕ *complement of* C. Forming this set amounts to eliminating the ϕ-worlds from each alternative in C.

Remark 3 Other than adding the minimal provisions to handle interrogatives, Definition 14 mimics the behavior of Definition 13 exactly. This does not make full use of the resources provided by defining updates on sets of alternatives rather than mere sets of worlds. An enhanced clause (6) illustrates this: $C[\Diamond\phi] = \{c' \in C \cup \{\bigcup C[\phi]\} \mid C[\phi] \neq \{\varnothing\}\}$. According to this semantics, $\Diamond\phi$ not only tests that there is at least one ϕ-world in $\bigcup C$, but also brings the ϕ-worlds 'into view' by adding the set containing them as an alternative in the output body of information and issues. This is similar to Yalcin's (2008) idea of modal resolution and accommodates the data he uses to motivate that idea. But it parallels exactly Ciardelli *et al.*'s (in press) model of *might*'s attentive content and Murray's (in press) related proposal about the structure of speech acts. These sophistications are not essential to the data discussed here I will not populate the formalism with them. But I ultimately view them as natural components of the overall view of inquiry and language use advocated here.

Definition 15 (Inquisitive State Semantics) $[\cdot] : (\mathcal{W}\!f\!f \times S) \mapsto S$
Where $s = \langle C, \langle C_0, \ldots \langle C_n \rangle \cdots \rangle\rangle$, $C = \{c_0, \ldots, c_n\}$, $\overline{C}_\phi := \{c_0 - \overline{c}_0, \ldots, c_n - \overline{c}_n\}$ and for $0 \leqslant i \leqslant n, \overline{c}_i := \bigcup X : \langle\{c_i\}\rangle[\phi] = \langle X, \ldots\rangle$:

$$
\begin{array}{lrcl}
(1) & s[\mathsf{p}] & = & \langle\{\, \{w \in c_0 \mid w(\mathsf{p}) = 1\}, \ldots, \{w \in c_n \mid w(\mathsf{p}) = 1\}\, \},\rangle\langle C_0, \ldots \langle C_n \rangle \cdots \rangle \\
(2) & s[\neg\phi] & = & \langle \overline{C}_\phi, \langle C_0, \ldots \langle C_n \rangle \cdots \rangle\rangle \\
(3) & s[\phi \wedge \psi] & = & s[\phi][\psi] \\
(4) & s[\phi \vee \psi] & = & \langle C' \cup C'', \langle C_0, \ldots \langle C_n \rangle \cdots \rangle\rangle \\
 & & & \text{where } s[\phi] = \langle C', \ldots\rangle \text{ and } s[\psi] = \langle C'', \ldots \rangle \\
(5) & s[?\phi] & = & \langle C' \cup \overline{C}_\phi, \langle C_0, \ldots \langle C_n \rangle \cdots \rangle\rangle \text{ where } s[\phi] = \langle C', \ldots\rangle \\
(6) & s[\Diamond\phi] & = & \langle\{c' \in C' \mid C' \neq \{\varnothing\}\}, \langle C_0, \ldots \langle C_n \rangle \cdots \rangle\rangle \text{ where } s[\phi] = \langle C', \ldots\rangle \\
(7) & s[\Box\phi] & = & \langle\{c' \in C' \mid C' = C\}, \langle C_0, \ldots \langle C_n \rangle \cdots \rangle\rangle \text{ where } s[\phi] = \langle C', \ldots\rangle
\end{array}
$$

Definition 16 (Inquisitive Conditional Semantics)

$$
s[(\mathsf{if}\,\phi)\,\psi] = \begin{cases} ((s \downarrow \mathsf{if}\,\phi) \Downarrow \phi) \uparrow \psi & \text{if } s[\phi] \neq \langle\{\varnothing\}, \ldots\rangle \\ \text{Undefined} & \text{otherwise} \end{cases}
$$

Remark 4 Definition 17 below assumes that each agent A's doxastic state in each world w may be modeled as a body of information and issues C_A^w which is a set of sets of worlds. Roughly put, the formula $\mathsf{W_A}(\mathsf{if}\,\phi)$ is true in w just in case A's information in w does not entail ϕ and A has distinguished ϕ and $\neg\phi$ as alternatives to decide between. The latter is guaranteed by requiring that updating the state $\langle C_A^w \rangle$ gives you back a state with the very same body of information and issues. Note, however, that $\mathsf{W_A}(\mathsf{if}\,\phi)$ presupposes that there is at least one ϕ-world in C_A^w. As I suggested in the main text, this presupposition originates with the *if*-clause and also surfaces in conditionals. The update format makes Definition 17 look much more complicated than what I just stated. In the Definition one must look at each alternative $c_i \in C$, eliminate any world w where A's information entails ϕ or A hasn't distinguished ϕ and $\neg\phi$ as alternatives to be decided between. Aloni & van Rooy's (2002: §3.6) semantics has much in common with this.

Definition 17 (Inquisitive Attitude Semantics)
Where $s = \langle C, \langle C_0, \ldots \langle C_n \rangle \cdots \rangle\rangle$ and $C = \{c_0, \ldots, c_n\}$:

$$s[\mathsf{W_A}(\mathsf{if}\,\phi)] = \begin{cases} \langle \{ \{w \in c_0 \mid \langle C_A^w \rangle[\mathsf{if}\,\phi] = \langle C_A^w, \ldots\rangle \;\&\; \langle C_A^w\rangle \not\vDash \phi\}, \\ \ldots, \{w \in c_n \mid \langle C_A^w \rangle[\mathsf{if}\,\phi] = \langle C_A^w, \ldots\rangle \;\&\; \langle C_A^w\rangle \not\vDash \phi\} \}, \\ \qquad \langle C_0, \ldots \langle C_n \rangle \cdots \rangle \rangle & \text{if } \forall w \in \bigcup C : \\ & \quad \langle C_A^w \rangle[\phi] \neq \langle \{\varnothing\}, \ldots\rangle \\ \text{Undefined} & \text{otherwise} \end{cases}$$

A.4 Semantic Concepts

Definition 18 (Informational Semantic Concepts)

(1) Support $c \vDash \phi \Leftrightarrow c[\phi] = c$ (2) Truth in w $w \vDash \phi \Leftrightarrow \{w\}[\phi] = \{w\}$

(3) Inconsistency $c[\phi] = \varnothing$ (4) Semantic Content $[\![\phi]\!] = \{w \mid w \vDash \phi\}$

(5) Contextual Content $[\![\phi]\!]_c = c[\phi]$

(6) Logical Necessity $\vDash \phi \Leftrightarrow \forall c : c[\phi] = c$

(7) Entailment $\phi_1, \ldots, \phi_n \vDash \psi \Leftrightarrow \forall c : c[\phi_1] \cdots [\phi_n] \vDash \psi$

Definition 19 (Inquisitive Semantic Concepts)

(1) Support $C \vDash \phi \Leftrightarrow \bigcup C = \bigcup(C[\phi])$

(2) Truth in w $w \vDash \phi \Leftrightarrow \bigcup(\{\{w\}\}[\phi]) = \{w\}$

(3) Inconsistency $C[\phi] = \{\varnothing\}$

(4) Informational Content $[\![\phi]\!] = \{w \mid w \vDash \phi\}$

(5) Logical Necessity $\vDash \phi \Leftrightarrow \forall C : \bigcup C = \bigcup(C[\phi])$

(6) Entailment $\phi_1, \ldots, \phi_n \vDash \psi \Leftrightarrow \forall C : C[\phi_1] \cdots [\phi_n] \vDash \psi$

Definition 20 (Semantic Concepts)

(1) Support $s \vDash \phi \Leftrightarrow s[\phi] = s'$ & $\bigcup C_s = \bigcup C_{s'}$

(2) Truth in w $w \vDash \phi \Leftrightarrow \langle\{\{w\}\}\rangle[\phi] = \langle C', \ldots\rangle$ and $\bigcup C' = \{w\}$

(3) Inconsistency $s[\phi] = \langle\{\varnothing\}, \ldots\rangle$

(4) Informational Content $[\![\phi]\!] = \{w \mid w \vDash \phi\}$

(5) Logical Necessity $\vDash \phi \Leftrightarrow \forall C : \bigcup C = \bigcup C'$, where $\langle C\rangle[\phi] = \langle C', \ldots\rangle$

(6) Entailment $\phi_1, \ldots, \phi_n \vDash \psi \Leftrightarrow \forall s : s[\phi_1] \cdots [\phi_n] \vDash \psi$

Definition 21 (Entailment v2)
$\phi_1, \ldots, \phi_n \vDash \psi \Leftrightarrow \forall s :$ if $s[\phi_1] \cdots [\phi_n][\psi]$ is defined, $s[\phi_1] \cdots [\phi_n] \vDash \psi$

Definition 22 (Logical Necessity v2)
$\vDash \phi \Leftrightarrow \forall C :$ if $\langle C\rangle[\phi]$ is defined, $\bigcup C = \bigcup C'$ where $\langle C\rangle[\phi] = \langle C', \ldots\rangle$.

A.5 Facts

A.5.1 Persistence and Preservation

Here I define two properties of modal formulae in LIC and show which modal formulae have which of the properties.

Definition 23 (Persistence) ϕ is persistent iff $c' \vDash \phi$ if $c \vDash \phi$ and $c' \subseteq c$. (I.e. ϕ's support persists after more information comes in.)

Fact 3 In general, $\Diamond\phi$ is not persistent. Take a c containing many worlds but only one ϕ-world w. Then $c \vDash \Diamond\phi$, but $c - \{w\} \subseteq c$ and $c - \{w\} \nvDash \Diamond\phi$.

Fact 4 $(\text{if } \phi)\, \psi$ is persistent if its constituents are. Suppose $c \vDash (\text{if } \phi)\, \psi$. Then $c[\phi][\psi] = c[\phi]$. If both ϕ and ψ are persistent and $c' \subseteq c$ then $c'[\phi][\psi] = c'[\phi]$, hence $c'[\phi] \vDash \psi$ and so $c' \vDash (\mathsf{if}\ \phi)\,\psi$. So $(\mathsf{if}\ \phi)\,\psi$ is persistent too.

Remark 1 $\Diamond\phi$ is equivalent to $\neg((\mathsf{if}\ \phi \vee \neg\phi)\,\neg\phi)$, so there are non-persistent formulae even in the $\Diamond$-free fragment.

Fact 5 $\Box\phi$ is persistent if ϕ is. Suppose $c \vDash \Box\phi$. Then $c \vDash \phi$. If $c' \subseteq c$ and ϕ is persistent, then $c' \vDash \phi$ and hence $c' \vDash \Box\phi$.

Definition 24 (Preservative) ϕ is preservative iff $c'[\phi] \subseteq c[\phi]$ if $c' \subseteq c$. (I.e. as information grows ϕ continues to exclude any worlds it previously did.)

Fact 6 $\Box\phi$ is not preservative. Consider a c such that $(c - c[\phi]) \neq \varnothing$. Then $c[\Box\phi] = \varnothing$. Let $c' = c[\phi]$. Then $c'[\Box\phi] = c'$. So $c' \subseteq c$ but $c'[\Box\phi] \nsubseteq c[\Box\phi]$.

Fact 7 $(\mathsf{if}\ \phi)\,\psi$ is not preservative. Consider a c such that $(c - c[\phi \wedge \neg\psi]) \neq \varnothing$. Then $c[(\mathsf{if}\ \phi)\,\psi] = \varnothing$. Let $c' = c - c[\phi \wedge \neg\psi]$. Then $c'[(\mathsf{if}\ \phi)\,\psi] = c'$. So $c' \subseteq c$ but $c'[(\mathsf{if}\ \phi)\,\psi] \nsubseteq c[(\mathsf{if}\ \phi)\,\psi]$.

A.5.2 Validities

Fact 8 (The Inquisitive Conditional is Strict over *c*)
Let s be a state and c_s the contextual possibilities in that state. Then the effect of $(\mathsf{if}\ \phi)\,\psi$ on c_s is identical to the following update just defined on c:

$$c[(\mathsf{if}\ \phi)\,\psi] = \begin{cases} \{w \in c \mid c[\phi] \vDash \psi\} & \text{if } c[\phi] \neq \varnothing \\ \text{Undefined} & \text{otherwise} \end{cases}$$

Proof Proceed by induction on the complexity of ϕ and ψ. First, suppose they are conditional free, i.e. in $\mathcal{W}\!f\!f_A$. If $s[\phi] = \langle\varnothing, \ldots\rangle$ both $s[(\mathsf{if}\ \phi)\,\psi]$ and $c[(\mathsf{if}\ \phi)\,\psi]$ are undefined. Suppose $s[\phi] \neq \langle\varnothing, \ldots\rangle$. Then $c[\phi] \neq \varnothing$. By Definitions 16 and 12.3 $s[(\mathsf{if}\ \phi)\,\psi] = \langle\{c \in C \mid \langle C\rangle[\phi] \vDash \psi\}, \ldots\rangle$. It is clear that $\bigcup(\{c \in C \mid \langle C\rangle[\phi] \vDash \psi\}) = \{w \in c \mid c[\phi] \vDash \psi\}$, and thus that $(\mathsf{if}\ \phi)\,\psi$'s effect on s_c is just the effect on c described by the Fact. Now suppose that ϕ and ψ may contain conditionals and grant the inductive hypothesis that Fact 8 holds for them. Suppose $s[\phi] \neq \langle\varnothing, \ldots\rangle$. Then by hypothesis $c[\phi] \neq \varnothing$ and by the same reasoning above, the Fact holds.

Remark 2 When proving a validity $\phi \vDash \psi$ below, I will follow Definition 21 and assume that $c[\phi][\psi]$ is defined. My goal will be to show that $c[\phi][\psi] = c[\phi]$.

Fact 9 (Modus Ponens) $(\mathsf{if}\ \phi)\,\psi, \phi \vDash \psi$
Proof Either $c[(\mathsf{if}\ \phi)\,\psi] = c$ or $c[(\mathsf{if}\ \phi)\,\psi] = \varnothing$. In the former case $c[(\mathsf{if}\ \phi)\,\psi][\phi][\psi] =$

$c[(\textsf{if}\,\phi)\,\psi][\phi]$ is equivalent to $c[\phi][\psi] = c[\phi]$, and it is also guaranteed that $c[\phi] \vDash \psi$ (by Fact 8). By the last point it follows that $c[\phi][\psi] = c[\phi]$ and hence by the equivalence that $c[(\textsf{if}\,\phi)\,\psi][\phi][\psi] = c[(\textsf{if}\,\phi)\,\psi][\phi]$. In the latter case $c[(\textsf{if}\,\phi)\,\psi][\phi][\psi] = \varnothing = c[(\textsf{if}\,\phi)\,\psi][\phi]$.

Fact 10 (Modus Tollens) For preservative ψ, $(\textsf{if}\,\phi)\,\psi, \neg\psi \vDash \neg\phi$
PROOF Suppose ψ is preservative. If the update with the conclusion is defined, the test imposed by the premise must be successful and so $c[\phi][\psi] = c[\phi]$. To show that the inference is valid, we must show that $c[\neg\psi] \vDash \neg\phi$. This amounts to $c[\neg\psi][\neg\phi] = c[\neg\psi]$. Since update is eliminative, $c[\neg\psi][\neg\phi] \subseteq c[\neg\psi]$. Hence we must show that $c[\neg\psi] \subseteq c[\neg\psi][\neg\phi]$. This simplifies to $(c - c[\psi]) - (c - c[\psi])[\phi] \subseteq c - c[\psi]$. For reductio, suppose w is not in the set named on the left, but is in the set named on the right. In virtue of the former fact $w \in (c-c[\psi])[\phi]$. If ϕ is preservative, it follows that $w \in c[\phi]$, since $c-c[\psi] \subseteq c$. Then it follows that $w \in c[\phi][\psi]$. Since ψ is preservative and $c[\phi] \subseteq c$, $w \in c[\psi]$. This is a contradiction since $w \in c - c[\psi]$. If ϕ isn't preservative, the only way for $(c-c[\psi])[\phi] \subseteq c[\phi]$ to fail is for $c[\phi] = \varnothing$ and $(c-c[\psi])[\phi] \neq \varnothing$. But this cannot occur since the premise presupposes that $c[\phi] \neq \varnothing$ and the presupposition is assumed to be met. Hence ϕ need not be preservative.

Remark 3 $(\textsf{if p})\,\Box\textsf{q}, \neg\Box\textsf{q} \nvDash \neg\textsf{p}$. Suppose we know that either (i) a and b are squares or (ii) a is a circle and b is a square or (iii) both a and b are circles. Then, if a is a square, b must be a square. Also, it's not the case that b must be a square. But it does not follow that a is not a circle.

Remark 4 $(\textsf{if p})\,((\textsf{if q})\,\textsf{r}), \neg(\textsf{if q})\,\textsf{r} \nvDash \neg\textsf{p}$. Suppose we know that either (i) a, b and c are squares or (ii) b is a square, but a and c are circles. So, if a is a square, then if b is a square c is too. Also, it's false that if b is a square, c is a square; after all b could be a square while c is a circle. Yet it does not follow that a is not a square.

Fact 11 (Contraposition) For preservative ψ, $(\textsf{if}\,\phi)\,\psi \vDash (\textsf{if}\,\neg\psi)\,\neg\phi$
PROOF Suppose ψ is preservative. If the update with the conclusion is defined, the test imposed by the premise must be successful and so $c[\phi][\psi] = c[\phi]$. To show that the inference is valid, we must show that $c[\neg\psi] \vDash \neg\phi$. This has already been shown to hold under these conditions in the proof of Fact 10.

Fact 12 (Identity) For persistent ϕ, $\vDash (\textsf{if}\,\phi)\,\phi$
PROOF By Fact 8, $c[(\textsf{if}\,\phi)\,\phi] = \{w \in c \mid c[\phi] \vDash \phi\} = \{w \in c \mid c[\phi][\phi] = c[\phi]\}$. Since ϕ is persistent $c[\phi][\phi] = c[\phi]$. So $c[(\textsf{if}\,\phi)\,\phi] = c$ and hence $\vDash (\textsf{if}\,\phi)\,\phi$.

Remark 5 Consider $(\mathsf{if}\ \Diamond \mathsf{p} \wedge \neg \mathsf{p})\,(\Diamond \mathsf{p} \wedge \neg \mathsf{p})$. This will amount to testing that $c[\Diamond \mathsf{p}][\neg \mathsf{p}] \vDash \Diamond \mathsf{p} \wedge \neg \mathsf{p}$. But this test will fail, since after taking in $\neg \mathsf{p}$ the information no longer supports the first conjunct $\Diamond \mathsf{p}$.

Fact 13 (Import-Export) $(\mathsf{if}\ \phi_1)\,((\mathsf{if}\ \phi_2)\,\psi) \dashv\vDash (\mathsf{if}\ \phi_1 \wedge \phi_2)\,\psi$

Proof

$$\begin{aligned} c[(\mathsf{if}\ \phi_1)\,((\mathsf{if}\ \phi_2)\,\psi)] &= \{w \in c \mid c[\phi_1] \vDash (\mathsf{if}\ \phi_2)\,\psi\} \\ &= \{w \in c \mid c[\phi_1][(\mathsf{if}\ \phi_2)\,\psi] = c[\phi_1]\} \\ &= \{w \in c \mid \{w' \in c[\phi_1] \mid c[\phi_1][\phi_2] \vDash \psi\} = c[\phi_1]\} \\ &= \{w \in c \mid c[\phi_1][\phi_2] \vDash \psi\} \\ &= \{w \in c \mid c[\phi_1 \wedge \phi_2] \vDash \psi\} \\ &= c[(\mathsf{if}\ \phi_1 \wedge \phi_2)\,\psi] \end{aligned}$$

Fact 14 (Ant. Strength.) For persistent ψ, $(\mathsf{if}\ \phi_1)\,\psi \vDash (\mathsf{if}\ \phi_1 \wedge \phi_2)\,\psi$

Proof If $c[(\mathsf{if}\ \phi_1)\,\psi][(\mathsf{if}\ \phi_1 \wedge \phi_2)\,\psi]$ is defined, $c[(\mathsf{if}\ \phi_1)\,\psi] = c$ and so $c[\phi_1] \vDash \psi$. $c[\phi_1][\phi_2] \subseteq c[\phi_1]$ and since ψ is persistent, $c[\phi_1][\phi_2] \vDash \psi$. Thus, $c[(\mathsf{if}\ \phi_1 \wedge \phi_2)\,\psi] = c$ and hence $c[(\mathsf{if}\ \phi_1)\,\psi][(\mathsf{if}\ \phi_1 \wedge \phi_2)\,\psi] = c[(\mathsf{if}\ \phi_1)\,\psi]$.

Remark 6 $(\mathsf{if}\ \Diamond \mathsf{p})\,\Diamond \mathsf{p} \nvDash (\mathsf{if}\ \Diamond \mathsf{p} \wedge \neg \mathsf{p})\,\Diamond \mathsf{p}$. Let c contain some p-world. Then $c[(\mathsf{if}\ \Diamond \mathsf{p})\,\Diamond \mathsf{p}] = c$. But $c \nvDash (\mathsf{if}\ \Diamond \mathsf{p} \wedge \neg \mathsf{p})\,\Diamond \mathsf{p}$, since $c[\Diamond \mathsf{p}][\neg \mathsf{p}] \nvDash \Diamond \mathsf{p}$. After all, $c[\Diamond \mathsf{p}][\neg \mathsf{p}][\Diamond \mathsf{p}] = \varnothing$ not c.

Fact 15 (Disj. Ants.) For persistent ψ, $(\mathsf{if}\ \phi_1 \vee \phi_2)\,\psi \vDash (\mathsf{if}\ \phi_1)\,\psi \wedge (\mathsf{if}\ \phi_2)\,\psi$

Proof The premise tests that $c[\phi_1] \cup c[\phi_2] \vDash \psi$. The conclusion presupposes that $c[\phi_1] \neq \varnothing$ and $c[\phi_2] \neq \varnothing$, and tests that $c[\phi_1] \vDash \psi$ and $c[\phi_2] \vDash \psi$. Since $c[\phi_1] \subseteq (c[\phi_1] \cup c[\phi_2])$ and $c[\phi_2] \subseteq (c[\phi_1] \cup c[\phi_2])$, this test must be successful when ψ is persistent but may not be successful when ψ isn't persistent.

Remark 7 $(\mathsf{if}\ \mathsf{p} \vee \neg \mathsf{p})\,\Diamond \mathsf{p} \nvDash ((\mathsf{if}\ \mathsf{p})\,\Diamond \mathsf{p}) \wedge ((\mathsf{if}\ \neg \mathsf{p})\,\Diamond \mathsf{p})$. If there are both p and $\neg$p worlds in c all presuppositions will be met and the premise will successfully test c. The second conjunct of the conclusion won't.

Fact 16 (Transitivity) For persistent ψ, $(\mathsf{if}\ \phi_1)\,\phi_2, (\mathsf{if}\ \phi_2)\,\psi \vDash (\mathsf{if}\ \phi_1)\,\psi$

Proof If $c[(\mathsf{if}\ \phi_1)\,\phi_2][(\mathsf{if}\ \phi_2)\,\psi][(\mathsf{if}\ \phi_1)\,\psi]$ is defined:

(A.1) $c[(\mathsf{if}\ \phi_1)\,\phi_2][(\mathsf{if}\ \phi_2)\,\psi] = c$.

Thus, we must show that $c[(\mathsf{if}\ \phi_1)\,\psi] = c$. By Fact 8 this amounts to showing that $c[\phi_1] \vDash \psi$, i.e. $c[\phi_1][\psi] = c[\phi_1]$. Fact 8 and (A.1) require $c[\phi_2] \vDash \psi$, hence:

(A.2) $c[\phi_2][\psi] = c[\phi_2]$

Assume ϕ_2 is preservative. Then, since $c[\phi_1] \subseteq c$:

(A.3) $c[\phi_1][\phi_2] \subseteq c[\phi_2]$

Together with ψ's persistence (A.2) and (A.3) entail that $c[\phi_1][\phi_2][\psi] = c[\phi_1][\phi_2]$.

But then $c[\phi_1][\psi] = c[\phi_1]$, since $c[\phi_1][\phi_2] = c[\phi_1]$. After all, Fact 8 and (A.1) require that $c[\phi_1] \vDash \phi_2$ and hence $c[\phi_1][\phi_2] = c[\phi_1]$. If ϕ_2 is not preservative the only way for $c[\phi_1][\phi_2] \subseteq c[\phi_2]$ to fail is for $c[\phi_2] = \varnothing$ and $c[\phi_1][\phi_2] \neq \varnothing$. But that can't happen since $(\mathsf{if}\,\phi_2)\,\psi$ presupposes that $c[\phi_2] \neq \varnothing$. Hence the argument above goes through without the assumption that ϕ_2 is preservative.

Remark 8 $(\mathsf{if}\,\neg\mathsf{p})\,\mathsf{q}, (\mathsf{if}\,\mathsf{q})\,\Diamond\mathsf{p} \nvDash (\mathsf{if}\,\neg\mathsf{p})\,\Diamond\mathsf{p}$. Let $c = \{w_0, w_1, w_2\}$, where w_0 is a $\mathsf{p} \wedge \mathsf{q}$-world, w_1 is a $\neg\mathsf{p} \wedge \mathsf{q}$-world and w_2 is a $\mathsf{p} \wedge \neg\mathsf{q}$-world. The first premise successfully tests c, since the only $\neg\mathsf{p}$-world in c is a q-world, namely w_1. The second premise is also successful since one of the q-worlds is a p-world, namely w_0. But the conclusion fails: among the $\neg\mathsf{p}$-worlds in c there are no p-worlds!

Fact 17 (Might) $(\mathsf{if}\,\phi)\,\Diamond\psi \dashv\vDash \Diamond(\phi \wedge \psi)$

PROOF

$$\begin{aligned} c[(\mathsf{if}\,\phi)\,\Diamond\psi] &= \{w \in c \mid c[\phi] \vDash \Diamond\psi\} \\ &= \{w \in c \mid c[\phi][\Diamond\psi] = c[\phi]\} \\ &= \{w \in c \mid c[\phi][\psi] \neq \varnothing\} \\ &= \{w \in c \mid c[\phi \wedge \psi] \neq \varnothing\} \\ &= c[\Diamond(\phi \wedge \psi)] \end{aligned}$$

Fact 18 (Must) $\Box(\mathsf{if}\,\phi)\,\psi \dashv\vDash (\mathsf{if}\,\phi)\,\psi \dashv\vDash (\mathsf{if}\,\phi)\,\Box\psi$

PROOF

$$\begin{aligned} c[\Box(\mathsf{if}\,\phi)\,]\psi &= \{w \in c \mid c[(\mathsf{if}\,\phi)\,\psi] = c\} \\ &= \{w \in c \mid c[\phi] \vDash \psi\} \\ &= c[(\mathsf{if}\,\phi)\,\psi] \\ &= \{w \in c \mid c[\phi][\psi] = c[\phi]\} \\ &= \{w \in c \mid c[\phi][\Box\psi] = c[\phi]\} \\ &= \{w \in c \mid c[\phi] \vDash \Box\psi\} \\ &= c[(\mathsf{if}\,\phi)\,\Box\psi] \end{aligned}$$

References

Adams, E. W. (1975). *The Logic of Conditionals*. Dordrecht: D. Reidel.

Aloni, M. & van Rooy, R. (2002). 'The Dynamics of Questions and Focus'. In Jackson, B., ed., 'Proceedings from Semantics and Linguistic Theory XII', 20–39. Ithaca, NY: Cornell University. URL http://staff.science.uva.nl/~maloni/salt02/salt02.pdf.

Appiah, A. (1985). *Assertion and Conditionals*. New York: Cambridge University Press.

Austin, J. L. (1956). 'Ifs and Cans'. *Proceedings of the British Academy* **42**: pp. 109–132.

Beaver, D. (2001). *Presupposition and Assertion in Dynamic Semantics*. Stanford, California: CSLI Publications.

Belnap, N. (1973). 'Conditional Assertion and Restricted Quantification'. In Leblanc, H., ed., 'Truth, Syntax and Modality', 48–75. Amsterdam: North-Holland Publishing Co.

Belnap, N. D. (1990). 'Declaratives Are Not Enough'. *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition* **59**(1): pp. 1–30. URL http://www.jstor.org/stable/4320114.

Bennett, J. (2003). *A Philosophical Guide to Conditionals*. Oxford: Oxford University Press.

van Benthem, J. (1996). *Exploring Logical Dynamics*. Stanford, California: CSLI Publications.

van Benthem, J., Gerbrandy, J. & Kooi, B. (2009). 'Dynamic update with probabilities'. *Studia Logica* **93**(1).

van Benthem, J., Muskens, R. & Visser, A. (1997). 'Dynamics'. In van Benthem, J. & ter Meulen, A., eds., 'Handbook of Logic and Language', 587–648. Amsterdam: Elsevier Science Publishers.

Bhatt, R. & Pancheva, R. (2006). 'Conditionals'. In Everaert, M. & van Riemskijk, H., eds., 'The Blackwell Companion to Syntax', vol. 1, chap. 16, 638–687. Malden, MA: Blackwell. URL http://www-rcf.usc.edu/~pancheva/bhatt-pancheva_syncom.pdf.

Bolinger, D. (1978). 'Yes–No Questions Are Not Alternative Questions'. In Hiz, H., ed., 'Questions', 87–105. Dordrecht: D. Reidel Publishing Company.

Caponigro, I. (2004). 'The Semantic Contribution of Wh-words And Type Shifts: Evidence from Free Relatives Crosslinguistically'. In Young, R. B., ed., 'Proceedings from Semantics and Linguistic Theory (SALT) XIV', 38–55. Ithaca, NY: CLC Publications. URL http://idiom.ucsd.edu/~ivano/Papers/2004_SALT%20XIV_revised%202008.pdf.

Chomsky, N. (1964). 'Current Issues in Linguistic Theory'. In Fodor, J. A. & Katz, J. J., eds., 'The Structure of Language', Englewood Cliffs, NJ: Prentice-Hall.

Ciardelli, I., Groenendijk, J. & Roelofsen, F. (in press). 'Attention! *Might* in Inquisitive Semantics'. In Ito, S. & Cormany, E., eds., 'Proceedings from Semantics and Linguistic Theory XIX', Ithaca, NY: CLC Publications.

Clark, H. H. (1996). *Using Language*. Cambridge, England: Cambridge University Press.

DeRose, K. (1991). 'Epistemic Possibilities'. *The Philosophical Review* **107**(2): pp. 581–605.

ECKARDT, R. (2007). 'The Syntax and Pragmatics of Embedded Yes/No Questions'. In SCHWABE, K. & WINKLER, S., eds., 'On Information Structure, Meaning and Form', 447–466. Amsterdam: John Benjamins. URL http://wwwuser.gwdg.de/~reckard/publikationen/EckardtIASG.pdf.

EDGINGTON, D. (1995). 'On Conditionals'. *Mind, New Series* **104**(413): pp. 235–329.

EDGINGTON, D. (2008). 'Conditionals'. In ZALTA, E. N., ed., 'The Stanford Encyclopedia of Philosophy (Winter 2008 Edition)', URL http://plato.stanford.edu/archives/win2008/entries/conditionals/.

EMBICK, D. & IATRIDOU, S. (1994). 'Conditional Inversion'. In GONZÁLES, M., ed., 'Proceedings of the North Eastern Linguistic Society 24', 189–203. Amherst, MA: Graduate Linguistics Association.

FAGIN, R., HALPERN, J. Y., MOSES, Y. & VARDI, M. Y. (1995). *Reasoning about Knowledge*. Cambridge, Massachusetts: The MIT Press.

DE FINETTI, B. (1936). 'La Logique de la Probabilité'. *Actes du Congrés International de Philosophie Scientifique* **4**: pp. 31–9.

VON FINTEL, K. (1999). 'The Presupposition of Subjunctive Conditionals'. In SAUERLAND, U. & PERCUS, O., eds., 'The Interpretive Tract', vol. MIT Working Papers in Linguistics 25, 29–44. Cambridge, MA: MITWPL. URL http://mit.edu/fintel/www/subjunctive.pdf.

VON FINTEL, K. & GILLIES, A. S. (2007). '*Might* Made Right'. Ms. MIT and Rutgers, URL http://semanticsarchive.net/Archive/TAyZGY2Z/mmr.pdf.

VON FINTEL, K. & GILLIES, A. S. (2008). 'Cia leaks'. *Philosophical Review* **117**(1). URL http://semanticsarchive.net/Archive/zczZDQ4N/cia_leaks.pdf.

FREGE, G. (1893). *Grundgesetze der Aritmetik, begriffsschriftlich abgeleitet, Vol. 1*. Jena: H. Pohle, 1st edn.

GIBBARD, A. F. (1981). 'Two Recent Theories of Conditionals'. In HARPER, W. L., STALNAKER, R. C. & PEARCE, G., eds., 'Ifs: Conditionals, Beliefs, Decision, Chance, Time', 211–247. Dordrecht: D. Reidel Publishing Co.

GILLIES, A. (2004). 'Epistemic Conditionals and Conditional Epistemics'. *Noûs* **38**(4): pp. 585–616. URL http://rci.rutgers.edu/~thony/epi_cond_nous.pdf.

GILLIES, A. (2009). 'On Truth-Conditions for *If* (but Not Quite Only *If*)'. *Philosophical Review* **118**(3): pp. 325–349. URL http://rci.rutgers.edu/~thony/truth-conditions-if-phil-review-2009.pdf.

GRICE, H. P. (1957). 'Meaning'. *Philosophical Review* **66**: pp. 377–388.

GRICE, H. P. (1989a). 'Indicative Conditionals'. In 'Studies in the Way of Words', chap. 4, 58–85. Cambridge, MA: Harvard University Press.

GRICE, H. P. (1989b). *Studies in the Way of Words*. Cambridge, Massachusetts: Harvard University Press.

Groenendijk, J. (1999). 'The Logic of Interrogation: Classical Version'. In Matthews, T. & Strolovitch, D., eds., 'Proceedings from Semantics and Linguistic Theory IX', 109–126. Ithaca, NY: Cornell University.

Groenendijk, J. & Stokhof, M. (1997). 'Questions'. In van Benthem, J. & ter Meulen, A., eds., 'Handbook of Logic and Language', 1055–1125. MIT Press. URL http://dare.uva.nl/document/3703.

Haiman, J. (1978). 'Conditionals are Topics'. *Language* **54**(3): pp. 564–589.

Hájek, A. & Hall, N. (1994). 'The Hypothesis of the Conditional Construal of Conditional Probability'. In Eells, E. & Skyrms, B., eds., 'Probability and Conditionals: Belief Revision and Rational Decision', 75–111. Cambridge, England: Cambridge University Press.

Hamblin, C. L. (1958). 'Questions'. *Australasian Journal of Philosophy* **36**: pp. 159–168.

Harman, G. (1975). 'Language, Thought, and Communication'. In Gunderson, K., ed., 'Language, Mind, and Knowledge. Minnesota Studies in the Philosophy of Science, Vol. 7', 270–298. Minneapolis, Minnesota: University of Minnesota Press.

Harman, G. (1979). '*If* and Modus Ponens'. *Theory and Decision* **11**: pp. 41–53.

Heim, I. (1982). *The Semantics of Definite and Indefinite Noun Phrases*. Ph.D. thesis, Linguistics Department, University of Massachusetts, Amherst, Massachusetts.

Heim, I. (1983). 'On the Projection Problem for Presuppositions'. In Barlow, M., Flickinger, D. P. & Wescoat, M. T., eds., 'Proceedings, Second West Coast Conference on Formal Linguistics', 114–125. Stanford, California: Stanford Linguistics Association.

Higginbotham, J. (1993). 'Interrogatives'. In Hale, K. & Keyser, S. J., eds., 'The View from Building 20: Essays in Linguistics in Honor of Sylvain Bromberger', 195–227. Cambridge, Massachusetts: The MIT Press.

Higginbotham, J. (1996). 'The Semantics of Questions'. In Lappin, S., ed., 'The Handbook of Contemporary Semantic Theory', 361–383. Oxford: Blackwell Publishers.

Hintikka, J. (1962). *Knowledge and Belief*. Ithaca, NY: Cornell University Press.

Huitink, J. (2008). *Modals, Conditionals and Compositionality*. Ph.D. thesis, University of Nijmegen, Nijmegen, Netherlands. URL http://ncs.ruhosting.nl/janneke/diss.html.

Jeffrey, R. C. (1963). 'On Indeterminate Conditionals'. *Philosophical Studies* **14**(3): pp. 37–43.

Jeffrey, R. C. (1990). *The Logic of Decision*. Chicago: University of Chicago Press, revised 2nd edn.

Jespersen, O. (1940). *A Modern English Grammar on Historical Principles, Part V: Syntax*, vol. 4. Copenhagen: Ejnar Munksgaard, 1st edn.

Karttunen, L. (1973). 'Presuppositions of Compound Sentences'. *Linguistic Inquiry* **4**: pp. 169–193. URL http://www2.parc.com/istl/members/karttune/publications/archive/plugsholesandfilters.pdf.

Kaufmann, S. (2000). 'Dynamic Context Management'. In Faller, M., Kaufmann, S. & Pauly, M., eds., 'Formalizing the Dynamics of Conversation', 171–188. Stanford, CA: CSLI Publications.

Kayne, R. (1991). 'Romance Clitics, Verb Movement and PRO'. *Linguistic Inquiry* **22**: pp. 647–686.

Kratzer, A. (1986). 'Conditionals'. In 'Proceedings from the 22nd Regional Meeting of the Chicago Linguistic Society', Chicago: University of Chicago. URL http://semanticsarchive.net/Archive/ThkMjYxN/Conditionals.pdf.

Kratzer, A. (1991). 'Modality'. In von Stechow, A. & Wunderlich, D., eds., 'Semantics: An International Handbook of Contemporary Research', 639–650. Berlin: de Gruyter.

Lewis, C. I. (1914). 'The Calculus of Strict Implication'. *Mind* **23**(90): pp. 240–247. URL http://www.jstor.org/stable/2248841.

Lewis, D. K. (1969). *Convention: A Philosophical Study*. Cambridge, Massachusetts: Harvard University Press.

Lewis, D. K. (1973). *Counterfactuals*. Cambridge, Massachusetts: Harvard University Press.

Lewis, D. K. (1975). 'Adverbs of Quantification'. In Keenan, E. L., ed., 'Formal Semantics of Natural Language', 3–15. Cambridge, England: Cambridge University Press.

Lewis, D. K. (1976). 'Probabilities of Conditionals and Conditional Probabilities'. *The Philosophical Review* **85**(3): pp. 297–315.

Lewis, D. K. (1979). 'Scorekeeping in a Language Game'. *Journal of Philosophical Logic* **8**(3): pp. 339–359.

Lewis, D. K. (1983). 'Individuation by Acquaintance and by Stipulation'. *The Philosophical Review* **92**(1): pp. 3–32. URL http://www.jstor.org/stable/2184519.

Lewis, D. K. (1986). 'Probabilities of Conditionals and Conditional Probabilities II'. *The Philosophical Review* **95**(4): pp. 581–589.

Lycan, W. (2006). 'Conditional-Assertion Theories of Conditionals'. In Thompson, J. & Byrne, A., eds., 'Content and Modality: Themes from the Philosophy of Robert Stalnaker', 148–164. New York: Oxford University Press. URL http://www.unc.edu/~ujanel/CondAssnThs.htm.

Lycan, W. G. (2001). *Real Conditionals*. Oxford: Oxford University Press.

Mackie, J. L. (1973). *Truth, Probability and Paradox*. Oxford: Oxford University Press.

McDermott, M. (1996). 'On the Truth Conditions of Certain 'If'-Sentences'. *The*

Philosophical Review **105**(1): pp. 1–37.

McGee, V. (1985). 'A Counterexample to Modus Ponens'. *Journal of Philosophy* **82**: pp. 462–471.

McGee, V. (1989). 'Conditional Probabilities and Compounds of Conditionals'. *Philosophical Review* **98**(4): pp. 485–541.

Milne, P. (1997). 'Bruno de Finetti and the Logic of Conditional Events'. *The British Journal for the Philosophy of Science* **48**(2): pp. 195–232.

Murray, S. E. (in press). 'A Hamblin Semantics for Evidentials'. In Ito, S. & Cormany, E., eds., 'Proceedings from Semantics and Linguistic Theory XIX', Ithaca, NY: CLC Publications. URL http://www.semanticsarchive.net/Archive/2I5ZjdmY/.

Muskens, R. (1996). 'Combining Montague Semantics and Discourse Representation'. *Linguistics and Philosophy* **19**(2): pp. 143–186.

Pratt, V. R. (1976). 'Semantical Considerations on Floyd-Hoare Logic'. In 'Proceedings of the 17th IEEE Symposium on Foundations of Computer Science', 109–121.

Pyers, J. & Emmorey, K. (2008). 'The Face of Bimodal Bilingualism'. *Psychological Science* **19**(6): pp. 531–6.

Quine, W. V. O. (1950). *Methods of Logic*. New York: Holt, Reinhart and Winston, 1st edn. References to 4th edition.

Ramsey, F. P. (1931). 'General Propositions and Causality'. In Braithwaite, R., ed., 'The Foundations of Mathematics: Collected Papers of Frank P. Ramsey', 237–255. London: Routledge and Kegan Paul.

Roberts, C. (2004). 'Context in Dynamic Interpretation'. In Horn, L. & Ward, G., eds., 'The Handbook of Pragmatics', 197–220. Malden, MA: Blackwell. URL http://www.ling.ohio-state.edu/~croberts/context.pdf.

Schaffer, J. (2004). 'From contextualism to contrastivism'. *Philosophical Studies* **119**(1-2): pp. 73–104.

Searle, J. (1969). *Speech Acts*. Cambridge, England: Cambridge Univesity Press.

Stalnaker, R. C. (1968). 'A Theory of Conditionals'. In Rescher, N., ed., 'Studies in Logical Theory', 98–112. Oxford: Basil Blackwell Publishers.

Stalnaker, R. C. (1975). 'Indicative Conditionals'. *Philosophia* **5**: pp. 269–286. Page references to reprint in Stalnaker (1999).

Stalnaker, R. C. (1976). 'Propositions'. In MacKay, A. & Merrill, D., eds., 'Issues in the Philosophy of Language', 79–91. New Haven: Yale University Press.

Stalnaker, R. C. (1999). *Context and Content: Essays on Intentionality in Speech and Thought*. Oxford: Oxford University Press.

Stalnaker, R. C. (2002). 'Common Ground'. *Linguistics and Philosophy* **25**(5–6): pp. 701–721.

Stalnaker, R. C. (2005). 'Conditional Propositions and Conditional Assertions'.

In Gajewski, J., Hacquard, V., Nickel, B. & Yalcin, S., eds., 'New Work on Modality', vol. 51 of *MIT Working Papers in Linguistics*. Cambridge, MA: MITWPL.

Strawson, P. F. (1952). *Introduction to Logical Theory*. London: Methuen.

Veltman, F. (1996). 'Defaults in Update Semantics'. *Journal of Philosophical Logic* **25**(3): pp. 221–261. URL http://www.pgrim.org/philosophersannual/xix/velt/index.htm.

Warmbrod, K. (1983). 'Epistemic Conditionals'. *Pacific Philosophical Quarterly* **64**: pp. 249–265. URL http://philpapers.org/archive/WAREC.1.pdf.

von Wright, G. H. (1957). *Logical Studies*. London: Routledge and Kegan Paul.

Yalcin, S. (2008). *Modality and Inquiry*. Ph.D. thesis, MIT, Department of Linguistics and Philosophy, Cambridge, MA. URL http://dspace.mit.edu/handle/1721.1/45893?show=full.