Expressing *May* and *Must*

Dynamic Semantics at Work

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Outline

The Role of Information

In Inquiry and Conversation

- Informational contents (*propositions*) are sets of possible worlds
  - These sets distinguish ways world might be (worlds in the set) from ways it isn't (worlds excluded from set)
- One informational content is particularly useful for understanding how linguistic interactions unfold:

Contextual Possibilities (c)

As communication and inquiry unfold, a body of information accumulates. Think of this information as what the agents are mutually taking for granted in some way. I call the set of worlds embodying this information c, short for *contextual possibilities*. (Stalnaker 1978; Lewis 1979)

Gaining Information

And Eliminating Possibilities

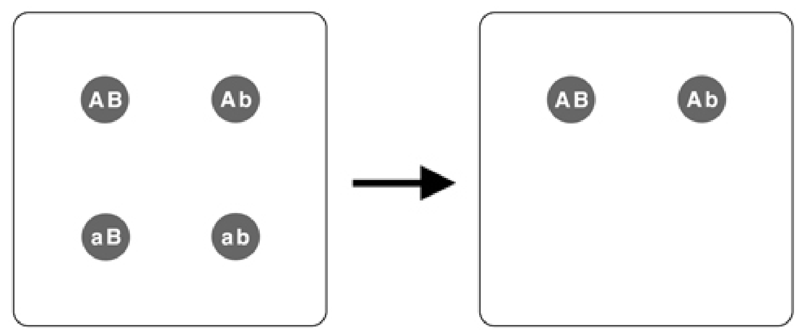

Figure: Accepting the information that A

- Inquiry progresses by using information to eliminate uncertainty, i.e. the elimination of worlds.
- $\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}, w_{\mathsf{aB}}, w_{\mathsf{ab}}\} \Rightarrow \{w_{\mathsf{AB}}, w_{\mathsf{Ab}}\}$

Question
How are Semantics and Information Change Related?

Classical Picture

1. **Semantics**: pair sentences w/propositions
   - $[\![\phi]\!]$ is a set of worlds
2. **Pragmatics**: rules for rational agents
   - When presented with information, rational agents use it to eliminate possibilities (update using intersection)

The Point: Semantics specifies informational content of a sentence, but nothing in particular about how sentence changes contextual information (c)

- Instead, pragmatics says how the sentence's content impacts c: $c \cap [\![\phi]\!]$

The Classical Picture
In More Detail

Classical Possible Worlds Semantics

1. $[\![\mathsf{A}]\!] = \{w \mid w(\mathsf{A}) = 1\}$
2. $[\![\neg\phi]\!] = W - [\![\phi]\!]$
3. $[\![\phi \wedge \psi]\!] = [\![\phi]\!] \cap [\![\psi]\!]$
4. $[\![\phi \vee \psi]\!] = [\![\phi]\!] \cup [\![\psi]\!]$
5. $[\![\Diamond\phi]\!] = \{w \mid \exists w' : w' \in Acc(w) \;\&\; w' \in [\![\phi]\!]\}$
   - $Acc(w)$ is the set of worlds accessible from w

Classical Truth and Consequence

Truth: $w \vDash \phi \iff w \in [\![\phi]\!]$

Consequence: $\phi \vDash \psi \iff [\![\phi]\!] \subseteq [\![\psi]\!]$

Two Important Consequences
Of the Classical Approach

Fact 1: $\Diamond \mathsf{A} \vDash \Diamond \mathsf{A} \vee \Diamond \mathsf{B}$

- Consequence is content inclusion:
  - $[\![\Diamond \mathsf{A}]\!] \subseteq [\![\Diamond \mathsf{A} \vee \Diamond \mathsf{B}]\!]$
- Disjunction is union:
  - $[\![\Diamond \mathsf{A}]\!] \subseteq [\![\Diamond \mathsf{A}]\!] \cup [\![\Diamond \mathsf{B}]\!]$

Fact 2: $\Diamond \mathsf{A} \vee \Diamond \mathsf{B} \nvDash \Diamond \mathsf{A}$

- This would require:
  - $[\![\Diamond \mathsf{A}]\!] \cup [\![\Diamond \mathsf{B}]\!] \subseteq [\![\Diamond \mathsf{A}]\!]$
  - But this only holds when $[\![\Diamond \mathsf{B}]\!] = \varnothing$

The First Problem of Free Choice
Kamp (1973) and Ross (1941)

(1) Billy may go to the beach

- Best classical approximation of *May*: some species of $\Diamond$
- May B $\overset{?}{=} \Diamond \mathsf{B}$

Problem: May B does not entail May B ∨ May C

X: Billy may go to the beach

Y: Ah, so Billy may go to the beach, or Billy may go the cinema × ×

Why is this a problem?

- Fact 1: $\Diamond \mathsf{B} \vDash \Diamond \mathsf{B} \vee \Diamond \mathsf{C}$

The Second Problem of Free Choice

Kamp (1973)

(2) Billy may go to the beach or he may go to the cinema

Problem: May B ∨ May C does entail May B

X: Billy may go to the beach or he may go to the cinema

Y: Ah, so Billy may go to the beach ✓

- Indeed: May B ∨ May C entails May B ∧ May C

Why is this a problem?

- Fact 2: $\Diamond A \vee \Diamond B \not\models \Diamond A$

- So there are at least two problems for classical approach to *may*

Tentative Diagnosis

The Performative Function of *May*

- A pragmatic approach...
  - Competence with a word is involved, so a semantic approach is preferable
  - Also, classical pragmatics assumes (declarative) sentences denote propositions
    - Proposition gets intersected with c
  - May A shouldn't change c, but rather Acc
  - This operation can't be intersection (sphere of accessibility should grow w/permission)
  - This would also require May A to denote an accessibility relation
    - So many problems w/this
- Alternative: approach to semantics that can directly encode (features of) communicative function of a word

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Dynamic Picture (Veltman, Heim)

1. **Semantics**: pair sentences with ways of changing contexts
   - ϕ eliminates worlds from c
2. **Pragmatics**: mechanisms for additional changes
   - General rationality, theory of mind, etc.

The Dynamic Picture

In More Detail

The Basic Idea

Assign each ϕ a function $[\phi]$ encoding how it changes c: $c[\phi] = c'$ (Better notation: $[\phi](c) = c'$)

Dynamic Informational Semantics

1. $c[\mathsf{A}] = \{w \in c \mid w(\mathsf{A}) = 1\}$
2. $c[\neg\phi] = c - c[\phi]$
3. $c[\phi \wedge \psi] = (c[\phi])[\psi]$
4. $c[\phi \vee \psi] = c[\phi] \cup c[\psi]$
5. $c[\Diamond\phi] = \{w \mid \exists w' : w' \in Acc(w)\ \&\ w' \in c[\phi]\}$
   - Veltman (1996): for epistemic *might*, $Acc(w)$ is c

Dynamic Informational Semantics

Semantic Concepts and a New Perspective on Classical Ones

Support, Consequence (Veltman)

- $c \Vdash \phi \iff c[\phi] = c$
- $\phi_1, \ldots, \phi_n \Vdash \psi \iff \forall c : c[\phi_1]\cdots[\phi_n] \Vdash \psi$

Truth, Propositions (Starr)

$w \vDash \phi \iff \{w\}[\phi] = \{w\} \qquad [\![\phi]\!] = \{w \mid w \vDash \phi\}$

Classical Consequence (Starr)

$\phi_1, \ldots, \phi_n \vDash \psi \iff \forall w : \{w\}[\phi_1]\cdots[\phi_n] \Vdash \psi$

- Classical logic is the logic of perfect information
- Equivalent to standard $\vDash$ for modal free fragment

Dynamic Informational Semantics

Understanding Classical vs. Dynamic Semantics

Corollary (Boolean Equivalence)

1. $[\![\mathsf{A}]\!] = \{w \mid w(\mathsf{A}) = 1\}$
2. $[\![\neg\phi]\!] = W - [\![\phi]\!]$
3. $[\![\phi \wedge \psi]\!] = [\![\phi]\!] \cap [\![\psi]\!]$
4. $[\![\phi \vee \psi]\!] = [\![\phi]\!] \cup [\![\psi]\!]$

- Booleans behave classically
- Boolean fragment: $\vDash$ and $\Vdash$ are identical
  - It's only with modals that $\vDash$ and $\Vdash$ come apart
- **Surprise**: $w \notin [\![\Diamond\phi]\!]$ unless $Acc(w) = \{w\}$ and $w \in [\![\phi]\!]$
- Then $[\![\Diamond\phi]\!] \subseteq [\![\phi]\!]$ and so $\Diamond\phi \vDash \phi$
  - Yet $\Diamond\phi \nVdash \phi$

Illustrating Our Surprise

$\Diamond\mathsf{A}$ is false unless A is true and $Acc(w) = \{w\}$

- Recall $w \in [\![\Diamond\mathsf{A}]\!] \iff \{w\}[\Diamond\mathsf{A}] = \{w\}$
- Consider whether $w_{\mathsf{A}} \in [\![\Diamond\mathsf{A}]\!]$:

$$\begin{aligned}\{w_{\mathsf{A}}\}[\Diamond\mathsf{A}] &= \{w \mid \exists w' : w' \in Acc(w) \;\&\; w' \in \{w_{\mathsf{A}}\}[\mathsf{A}]\}\\ &= \{w \mid w_{\mathsf{A}} \in Acc(w)\}\end{aligned}$$

- Unless $Acc(w_{\mathsf{A}}) = \{w_{\mathsf{A}}\}$, $\{w_{\mathsf{A}}\}[\Diamond\mathsf{A}] \neq \{w_{\mathsf{A}}\}$
- Consider whether $w_{\mathsf{a}} \in [\![\Diamond\mathsf{A}]\!]$

$$\begin{aligned}\{w_{\mathsf{a}}\}[\Diamond\mathsf{A}] &= \{w \mid \exists w' : w' \in Acc(w) \;\&\; w' \in \{w_{\mathsf{a}}\}[\mathsf{A}]\}\\ &= \{w \mid \exists w' : w' \in Acc(w) \;\&\; w' \in \varnothing\}\\ &= \varnothing\end{aligned}$$

- Nope! So $[\![\Diamond\mathsf{A}]\!] \subseteq [\![\mathsf{A}]\!]$

Classical Logic as Logic of Omniscience

A Logic of Omniscience is Not Suited for Modality

d'Alembert (1751) on Truth

"The universe... would only be one fact and one great truth for whoever knew how to embrace it from a single point of view." (d'Alembert 1995: 29)

- A sentence is true iff it corresponds w/'the great truth'
  - Correspond: cohere w/the great truth, $\{w\}[\phi] = \{w\}$
- But modal sentences are indexical to **uncertainty** or **utilities** of agent(s) evaluating them
- Whether they cohere with the great truth is irrelevant
- We want an evaluative concept that targets how they cohere with **partial information**: $c[\phi] = c$ (support)

Dynamic Informational Semantics

Back to Free Choice

- This system is quite useful for epistemic modals
  - $\mathsf{A} \wedge \Diamond\neg\mathsf{A}$ is inconsistent (cf. Veltman and Yalcin)
  - $\Diamond\phi$ can add possibilities to c
    - Not possible to get with $c \cap [\![\Diamond\phi]\!]$
- But it does not solve the problem of free choice:
  - $\Diamond\phi \not\Vdash \Diamond\phi \vee \Diamond\psi$
  - $\Diamond\phi \vee \Diamond\psi \not\Vdash \Diamond\phi$
- However, it is the backbone of a system that does:
  - *may*'s meaning resides in how it changes context
  - Use of $\Vdash$ rather than $\vDash$

Basic Idea

Capture the non-informational impact of *may* by sophisticating the contexts sentences operate on

Preference, Rationality & Context

Information and the Process of Inquiry

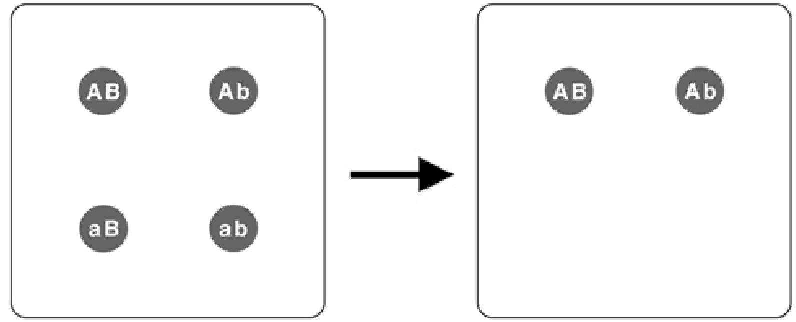

Figure: Accepting the information that A

- Inquiry progresses by gaining information, i.e. the elimination of worlds.
- $\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}, w_{\mathsf{aB}}, w_{\mathsf{ab}}\} \Rightarrow \{w_{\mathsf{AB}}, w_{\mathsf{Ab}}\}$

Preference, Rationality & Context

Issues

- It's not just information that accumulates in communication and inquiry (Bromberger 1966)
- There are issues (e.g. Hamblin 1958; Roberts 1996).
- They can be thought of as ways of grouping worlds in c into competing alternative propositions.

Alternatives (C) (e.g. Groenendijk 1999)

Alternatives represent open, competing propositions the agents are concerned with deciding between; their **issues**. Formally, this grouping of c may be identified with a set of sets of worlds; call it C. There is no need to also keep track of c: it is just the union of all the alternatives in C.

Preference, Rationality & Context

Issues and Inquiry

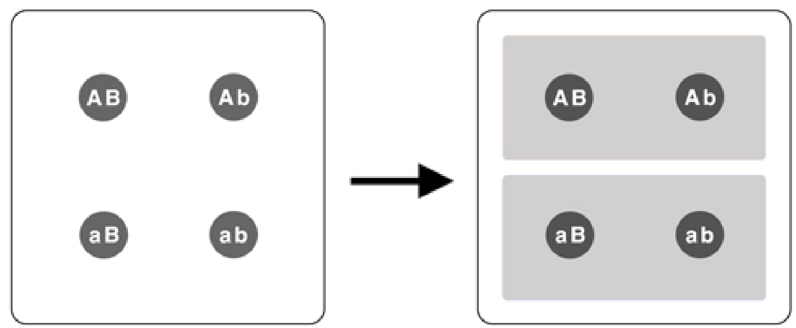

Figure: Recognizing the issue whether A

- Inquiry also progresses by recognizing issues, i.e. introducing alternatives
- $\{\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}, w_{\mathsf{aB}}, w_{\mathsf{ab}}\}\} \Rightarrow \{\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}\}, \{w_{\mathsf{aB}}, w_{\mathsf{ab}}\}\}$

Preference, Rationality & Context

Preferences

- Agents not only gather information and identify competing alternatives, they form **preferences** regarding those alternatives
- Central to **decision theoretic** approaches to rational choice, as applied in philosophy, AI and economics (e.g. Ramsey 1931; Newell 1992)
- Of relevance here: the preferences being mutually taken for granted for the purposes of an interaction
  - Parallel to Stalnaker's common ground

Preference, Rationality & Context

Preferences

- A body of preferences can be represented as a binary **preference relation** on the alternatives
- I.e. a set of pairs of propositions constructed from c

Preference State (R)

- R: binary relation on alternatives (open propositions)
- $R(a, a')$: a is preferred to a'
- Each pair in R is called a *preference*
- Set of (non-empty) alternatives over which R is defined: issues at stake in R, C_R
- Set of worlds among those alternatives: the contextual possibilities written c_R

Preference, Rationality & Context

Information in a Preference State

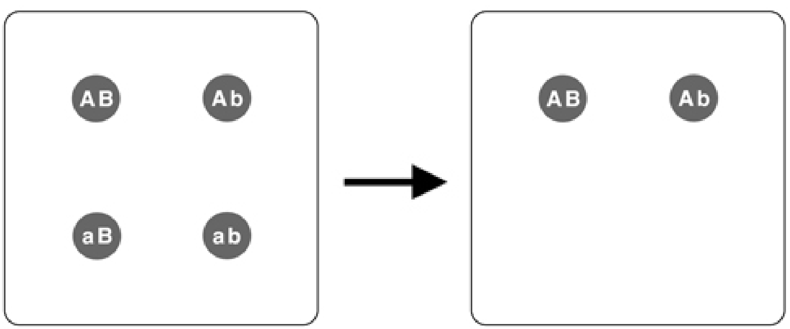

Figure: Accepting the information that A

- $\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}, w_{\mathsf{aB}}, w_{\mathsf{b}}\} \Rightarrow \{w_{\mathsf{AB}}, w_{\mathsf{Ab}}\}$
- $\{\langle\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}, w_{\mathsf{aB}}, w_{\mathsf{b}}\}, \varnothing\rangle\} \Rightarrow \{\langle\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}\}, \varnothing\rangle\}$

Preference, Rationality & Context

Issues in a Preference State

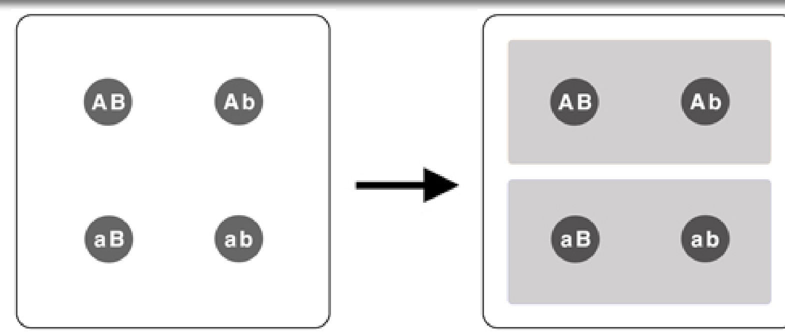

Figure: Recognizing the issue whether A

- $\{\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}, w_{\mathsf{aB}}, w_{\mathsf{ab}}\}\} \Rightarrow \{\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}\}, \{w_{\mathsf{aB}}, w_{\mathsf{ab}}\}\}$
- $\{\langle\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}, w_{\mathsf{aB}}, w_{\mathsf{ab}}\}, \varnothing\rangle\}$
  $\Rightarrow \{\langle\{w_{\mathsf{AB}}, w_{\mathsf{Ab}}\}, \varnothing\rangle, \langle\{w_{\mathsf{aB}}, w_{\mathsf{ab}}\}, \varnothing\rangle\}$

Preference, Rationality & Context

Preference and Inquiry

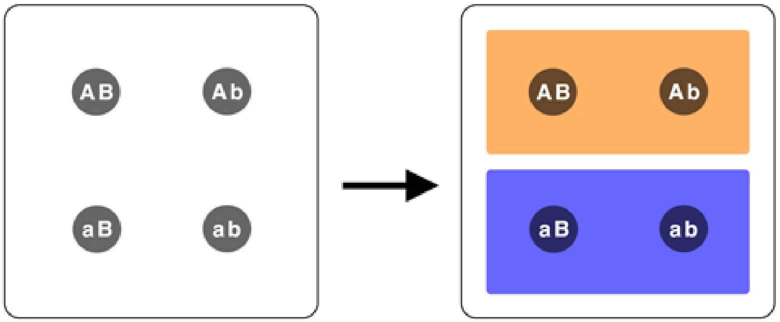

Figure: Coming to prefer A (to ¬A)

- $\{ \langle \{ w_{\mathsf{AB}}, w_{\mathsf{Ab}}, w_{\mathsf{aB}}, w_{\mathsf{ab}} \}, \varnothing \rangle \}$
  $\Rightarrow \{ \langle \{ w_{\mathsf{AB}}, w_{\mathsf{Ab}} \}, \{ w_{\mathsf{aB}}, w_{\mathsf{ab}} \} \rangle \}$

Preference, Rationality & Context

Preference and Inquiry

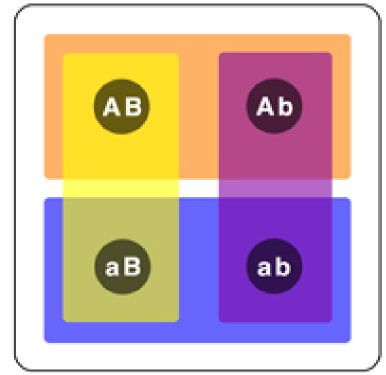

Figure: A more complex preference involving A and B

$\{ \langle \{ w_{\mathsf{AB}}, w_{\mathsf{Ab}} \}, \{ w_{\mathsf{aB}}, w_{\mathsf{ab}} \} \rangle, \langle \{ w_{\mathsf{AB}}, w_{\mathsf{aB}} \}, \{ w_{\mathsf{Ab}}, w_{\mathsf{ab}} \} \rangle \}$

Preference, Rationality & Context

Using Preference to Make Rational Choices

- Given preference relation, which alternatives are best?
- How do you use preferences to decide what to do?
- In decision theory, this takes the form of defining a choice function (Hansson & Grüne-Yanoff 2009)
- A choice function Ch maps a preference state R to the set of best alternatives according to R

Proposal: Choice, Permission, Requirement

1. $Ch(R)$ are the alternatives permissible according to R
2. Required by R: unique alternative permitted by R
   - Not always the case!

Preference, Rationality & Context

The Choice Function: Logical Weak Dominance

Which Alternatives are Best?

1. Competition between **preferred alternatives** $P(R)$
   - Left member in some pair
2. If preferred alternative a is entailed another preferred one, then a is out
3. If a entails a dispreferred alternative, a is out

Choice: Formally

$$Ch(R) = \{ a \in P(R) \mid \nexists a' \in P(R) : a' \subset a \;\; \& \; \nexists a' \in D(R) : a \subseteq a' \}$$

[$D(R)$: dispreferred alternatives]

Preference, Rationality & Context

How Choice Works: An Example

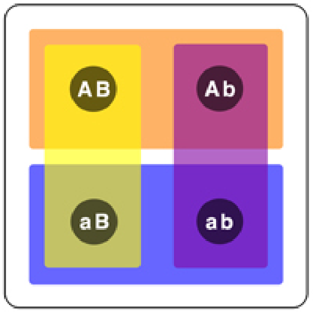

Figure: Preference for A with (separate) preference for B

- $\{ \langle \{w_{AB}, w_{Ab}\}, \{w_{aB}, w_{ab}\} \rangle, \langle \{w_{AB}, w_{aB}\}, \{w_{Ab}, w_{ab}\} \rangle \}$
- Two **preferred** (warm) alternatives, orange and yellow
- Neither entails the other nor dispreferred (cold) alt.
- So $Ch(R) = \{\{w_{AB}, w_{Ab}\}, \{w_{AB}, w_{aB}\}\}$

Preference, Rationality & Context

How Choice Works: A More Complex Example

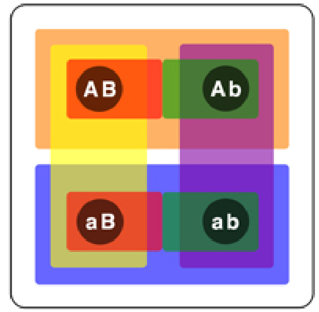

Figure: Pref A and B

- 4 pref. alt's: yellow, orange, reds
- Yellow is out: reds entail it
- Orange is out: top red entails it
- Bottom red is out: it entails blue, which is a dispreferred alt
- Unique best alternative: top red
- A ∧ B is required

$$\{ \langle \{w_{AB}, w_{Ab}, w_{aB}, w_{ab}\}, \varnothing \rangle, \langle \{w_{AB}, w_{Ab}\}, \{w_{aB}, w_{ab}\} \rangle, \langle \{w_{AB}\}, \{w_{Ab}\} \rangle, \langle \{w_{aB}\}, \{w_{ab}\} \rangle, \langle \{w_{AB}, w_{aB}\}, \{w_{Ab}, w_{ab}\} \rangle \}$$

The Semantics: in brief

The New Approach

- Dynamic Meaning: function from contents to contents
  - Now contents are preference states
- $R[\phi] = R'$: R' is the result of applying ϕ to R

The Basics

1. May ϕ tests that ϕ is consistent w/some $a \in Ch(R)$
   - If so, a preference for ϕ is added to R
     - This doesn't guarantee that ϕ will become $Ch(R)$
   - Otherwise: fail, i.e. return $\{\langle \varnothing, \varnothing \rangle\}$
2. Disjunction unions separate updates
3. Conjunction sequences updates

The Semantics Applied

How May Works: An Example

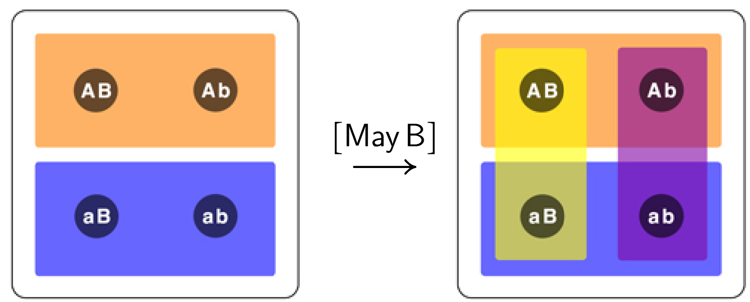

Figure: R_0 to R_0[May B]

- The best alternative (orange) is consistent w/B
- A preference and permission for B are introduced
- But a requirement for B is not

The Semantics Applied

How *May* Works: With Disjunction

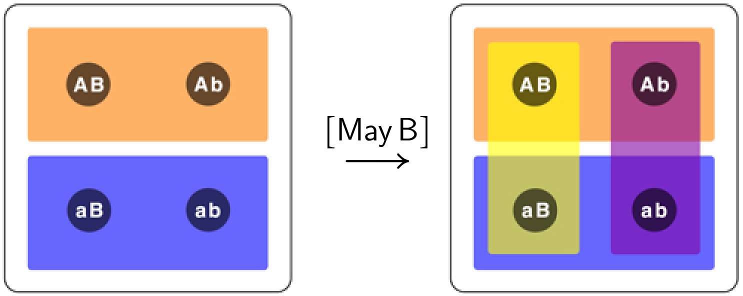

Figure: R_0 to R_0[May B]

- R_0 would be the result of R_0[May A]
- R_0[May A ∨ May B] = R_0[May A] ∪ R_0[May B] = R_0[May B]
- May A ⊮ May A ∨ May B? May A ∨ May B ⊩ May A?

Consequence

The Old and the New

Support, Consequence (Veltman)

- $c \Vdash \phi \iff c[\phi] = c$
- $\phi_1, \ldots, \phi_n \Vdash \psi \iff \forall c : c[\phi_1]\cdots[\phi_n] \Vdash \psi$

Preferential Support, Consequence (Starr)

- $R \Vdash \phi \iff Ch(R) = Ch(R[\phi])$
- $\phi_1, \ldots, \phi_n \Vdash \psi \iff \forall R : R[\phi_1]\cdots[\phi_n] \Vdash \psi$

- Both kinds of consequence and support are useful
- The first when tracking information
- The second when tracking the best alternatives

The First Problem of Free Choice

Modal Disjunction Intro is Invalid!

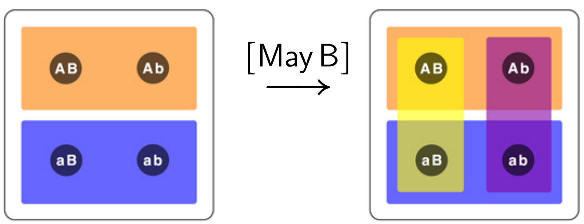

Figure: R_0 to R_0[May B] = R_0[May A ∨ May B]

- May A ⊩ May A ∨ May B means:

  $Ch(R_0$[May A]) = $Ch(R_0$[May A][May A ∨ May B])

  $Ch(R_0) = Ch(R_0$[May B])

- But yellow is in $Ch(R_0$[May B]), and not $Ch(R_0)$
- So May A ⊮ May A ∨ May B!

The Second Problem of Free Choice

From Disjunction to Conjunction

- Disjunctions are felicitous only when each disjunct compatible w/context (Stalnaker 1975: §III)
  - If $\phi \vee \psi$ is appropriate: $R[\phi] \neq \{\langle\varnothing,\varnothing\rangle\} \neq R[\psi]$
- What happens when ϕ is May A?
- If May A is compatible w/context then context passes its test and *may* adds preference for A
  - So R[May A] will be R plus a preference for A
- Same holds if ψ is May B
- Since R[May A ∨ May B] = R[May A] ∪ R[May B]:
  - Resulting state will have pref for A and for B and both will be consistent with best alternatives
- So after updating with May A ∨ May B, May A will not change the best alternatives

The Second Problem of Free Choice

From Disjunction to Conjunction

Important Observation

After updating with $\mathsf{May\,A} \vee \mathsf{May\,B}$, $\mathsf{May\,A}$ will not change the best alternatives

- The same holds for $\mathsf{May\,B}$

- But that is just to say:
  - $\mathsf{May\,A} \vee \mathsf{May\,B} \Vdash \mathsf{May\,A}$
  - $\mathsf{May\,A} \vee \mathsf{May\,B} \Vdash \mathsf{May\,B}$
  - $\mathsf{May\,A} \vee \mathsf{May\,B} \Vdash \mathsf{May\,A} \wedge \mathsf{May\,B}$
- Yet in the modal free fragment, $\vee$ and $\wedge$ behave exactly as the Boolean operators of classical semantics!
- **Proviso**: Stalnaker's observation treated as presupposition of $\vee$ and $\Vdash$ is Strawsonian

Free Choice and Two Views of Semantics Dynamic Preference Semantics References

The Semantics

In Full Detail

Atomic Semantics

- Where $R = \{\langle a_0, a_1\rangle, \ldots, \langle a_n, a_{n+1}\rangle\}$:
  $R[\mathsf{A}] = \{\langle a_0[\mathsf{A}], a_1[\mathsf{A}]\rangle, \ldots, \langle a_n[\mathsf{A}], a_{n+1}[\mathsf{A}]\rangle\}$

Connective Semantics

- $R[\phi \wedge \psi] = (R[\phi])[\psi]$
- $R[\phi \vee \psi] = \begin{cases} R[\phi] \cup R[\psi] & \text{if } R[\phi] \neq \{\langle\varnothing,\varnothing\rangle\} \neq R[\psi] \\ \text{Undefined} & \text{otherwise} \end{cases}$

Free Choice and Two Views of Semantics Dynamic Preference Semantics References

The Semantics

And Strawsonian Consequence

Modal Semantics

$$R[\mathsf{May}\,\phi] = \begin{cases} R \cup \{\langle c_R[\phi], c_R - c_R[\phi]\rangle\} & \text{if } \exists a \in Ch(R) : a[\phi] \neq \varnothing \\ \{\langle\varnothing,\varnothing\rangle\} & \text{otherwise} \end{cases}$$

$$R[\mathsf{Must}\,\phi] = \begin{cases} R \cup \{\langle c_R[\phi], c_R - c_R[\phi]\rangle\} & \text{if } \forall a \in Ch(R) : a[\phi] = a \\ \{\langle\varnothing,\varnothing\rangle\} & \text{otherwise} \end{cases}$$

Strawsonian Preferential Support, Consequence

- $R \Vdash \phi \iff R[\phi]$ is defined & $Ch(R) = Ch(R[\phi])$
- $\phi_1, \ldots, \phi_n \Vdash \psi \iff \forall R : R[\phi_1]\cdots[\phi_n] \Vdash \psi$ if $R[\phi_1]\cdots[\phi_n][\psi]$ is defined

Free Choice and Two Views of Semantics Dynamic Preference Semantics References

In Related Work

Imperatives

- Free choice behavior of *might*?
  - Veltman's semantics + this implementation of Stalnaker's observation
- Why not just use Veltman's semantics for *may*?
  - *might* has no 'deontic reading'
  - Preference semantics captures deontic reading of *may*
  - Epistemic reading of *may*?
- In other work I use dynamic preference semantics to analyze imperatives
  - Having an analysis of deontic modals allows me to chart their connections
  - E.g. $!\mathsf{A} \Vdash \mathsf{May\,A}$

Conclusions

About Dynamic Semantics

Summary

1. Classical semantics is a fragment of dynamic semantics
2. But expanding into the resources afforded only by dynamic semantics is fruitful
   - Here: free choice
3. These provide something like an expressivist semantics
   - Truth is not the only useful evaluative concept
   - Consequence is not always about truth
4. But avoids typical pitfalls
   - How do you blend the truth-conditional and the non-truth-conditional? (Frege-Geach)
   - DS: truth-conditional behavior emerges for limited fragment from more basic semantic constraints

Thank you!

(Slides available at `http://williamstarr.net/research`)

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